Decoherence and the Measurement Problem

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The problem of measurement

2 contradictory postulates:

- The Schrödinger equation: $i\hbar \frac{d\Psi}{dt} = H\Psi$

- The reduction postulate: $\Psi = \sum c_i |\varphi_i\rangle \rightarrow \varphi_k$

The problem of measurement

The measurement process: 2 points of view

System S: $|\Psi_S\rangle = \sum c_i |\varphi_i\rangle$. Apparatus A: $|A_0\rangle$

Grand system state product: $\Psi_{SA} = |\Psi_S\rangle|A_0\rangle = \sum c_i |\varphi_i\rangle|A_0\rangle$

After the interaction between S and A

- Schrödinger equation: $\Psi_{SA} = \sum c_i |\varphi_i\rangle|A_0\rangle \rightarrow \sum c_i |\varphi_i\rangle|A_i\rangle$
- Reduction postulate: $|\Psi_S\rangle = \sum c_i |\varphi_i\rangle \rightarrow \varphi_k$ and $|A_0\rangle \rightarrow |A_k\rangle$

The problem of measurement

A logical inconsistency inside the quantum formalism ?

- The role of consciousness (London & Bauer, Wigner)
- Modifying the Schrödinger Equation (G.R.W. formalism)

The problem of measurement

No system is really totaly isolated

Zeh(1970)

Take the Environment into account !

Decoherence : Zurek (1981)

The Decoherence Process

The density matrix formalism

$$\varrho_S = |\Psi_S\rangle\langle\Psi_S|$$

Pure case: $|\Psi_S\rangle = \alpha\,|\varphi_1\rangle + \beta\,|\varphi_2\rangle$

In the basis $(|\varphi_1\rangle, |\varphi_2\rangle)$: $\varrho_S = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$

$$\varrho_S{}^2 = \varrho_S$$

The Decoherence Process

The density matrix formalism

Statistical mixture: $|\Psi_k\rangle$ *with probability* p_k

$$|\Psi_k\rangle = \alpha_k \, |\varphi_1\rangle + \beta_k \, |\varphi_2\rangle$$

$$\varrho_k = |\Psi_k\rangle\langle\Psi_k| \quad \rho = \sum_k p_k \, \rho_k$$

$$\varrho_S{}^2 \neq \varrho_S$$

The Decoherence Process

The density matrix formalism

In two dimensions: Proper mixture of $|\varphi_1\rangle$ with probability p_1 and $|\varphi_2\rangle$ with probability p_2

In the basis $(|\varphi_1\rangle, |\varphi_2\rangle)$: $\varrho_S = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}$

$$\varrho_S{}^2 \neq \varrho_S$$

The Decoherence Process

The density matrix formalism

Interpretation:

The diagonal element ρ_{ii} is the probability to find the system in the state $|\varphi_i\rangle$

The non diagonal element ρ_{ij} is linked to the interference between $|\varphi_i\rangle$ and $|\varphi_j\rangle$

The Decoherence Process

The density matrix formalism

The Schrödinger equation gives:

$$i\hbar \frac{d}{dt}\varrho(t) = [H(t), \varrho(t)]$$

The Decoherence Process

The density matrix formalism

During a measurement (an interaction between S and A):
After the interaction, according to the Schrödinger equation:

$$\Psi_{SA} = \sum c_i \left|\varphi_i\right\rangle |A_0\rangle \rightarrow \sum c_i \left|\varphi_i\right\rangle |A_i\rangle$$

$$\varrho_{SA} = |\Psi_{SA}\rangle\langle\Psi_{SA}|$$

In two dimensions:

$$|\Psi_S\rangle = \alpha \left|\varphi_1\right\rangle + \beta \left|\varphi_2\right\rangle \text{ and } \Psi_{SA} = \alpha \left|\varphi_1\right\rangle |A_1\rangle + \beta \left|\varphi_2\right\rangle |A_2\rangle$$

In the basis ($\left|\varphi_1\right\rangle |A_1\rangle$, $\left|\varphi_2\right\rangle |A_2\rangle$): $\varrho_{SA} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$

The Decoherence Process

The role of the environment

$$\varrho_{SAE} = |\Psi_{SAE}\rangle\langle\Psi_{SAE}|$$

After the interaction, according to the Schrödinger equation:

$$\Psi_{SAE} = \sum c_i \left|\varphi_i\right\rangle|A_0\rangle|E_0\rangle \rightarrow \sum c_i|\varphi_i\rangle|A_i\rangle\,|E_i\rangle$$

In the basis ($\left|\varphi_1\right\rangle|A_1\rangle|E_1\rangle$, $\left|\varphi_2\right\rangle|A_2\rangle|E_2\rangle$): $\varrho_{SAE} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$

The Decoherence Process

The role of the environment

$$\varrho_{SAE} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$ What have we gained ? Apparently nothing!

The key point is:

We can't perform measurements on all the degrees of freedom of the environment

The Decoherence Process

The role of the environment

The subsystem SA is described by the density matrix got from the partial trace on the degrees of freedom of the environment of ϱ_{SAE} :

$$Tr_E \varrho_{SAE} = \varrho_{SA} = \begin{pmatrix} |\alpha|^2 & Z\alpha\beta^* \\ Z\alpha^*\beta & |\beta|^2 \end{pmatrix}$$

The Decoherence Process

The role of the environment

Now $Z(t) \to 0$ very rapidly. So

$$\varrho_{SA}(t) = \begin{pmatrix} |\alpha|^2 & Z(t)\alpha\beta^* \\ Z(t)\alpha^*\beta & |\beta|^2 \end{pmatrix} \to \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

This looks like the density matrix of a statistical mixture

The Decoherence Process

The measurement problem

What is got : An explanation of the classical appearance of the world, provided we use the standard recipes to compute. We can't observe any macroscopic superposition and the pointer basis is selected

The Decoherence Process

The measurement problem

Is it solved ? NO

1) The non diagnonal terms become big again (after a –very- long time)
2) The diagonal density matrix got from the partial trace is not the density matrix of a proper mixture
3) No individual system can have a diagonal density matrix
4) It remains to explain why only one value is selected and why the final state is reduced ("and/or" problem)

The Decoherence Process

The measurement problem

What is got : An explanation of the classical appearance of the world, provided we use the standard recipes to compute. We can't observe any macroscopic superposition and the pointer basis is selected

But the underlying reality (if there is any) remains in a superposed and intricated state

That would mean that the reduction postulate is nothing but a convenient way to describe the observations but doesn't correspond to any real physical process

The Decoherence Process

The measurement problem

The standard recipes to compute assume that we know what a measurement is

It is when a measurement is made that the probability of finding a specific result is given by the corresponding diagonal element of the density matrix

But nowhere inside the formalism of decoherence it is said what a measurement is. We are left with the initial problem!

The Decoherence Process

The measurement problem

What is a measurement ?

A dynamical process if the Schrodinger equation is modified: e.g. the G.R.W. formalism

If not: A measurement is made when an observation is made

Only consistent possibility ⇒ when a couscious mind becomes aware of the result

Real reduction ⇒ Action of mind on matter action (London and Bauer / Wigner)

The Decoherence Process

The measurement problem

What is a measurement ?

Reduction postulate or Everett Interpretation ?

No reduction: Everett Interpretation -> Convivial Solipsism

The Decoherence Process

The measurement problem

Everett interpretation: Relative States / Many Worlds / Many Minds

No reduction (the physical world remains in a superposed state) but the observer is divided

$$\Psi_{SAEO} = \sum c_i \left|\varphi_i\right\rangle \left|A_0\right\rangle \left|E_0\right\rangle \left|O_0\right\rangle \rightarrow \sum c_i \left|\varphi_i\right\rangle \left|A_i\right\rangle \left|E_i\right\rangle \left|O_i\right\rangle$$

There are as many observers as there are branches (which is not very economical)

The Decoherence Process

The Convivial Solipsism

The Hanging up mechanism:

$$\Psi_{SAEO} = \sum c_i \left|\varphi_i\right\rangle \left|A_0\right\rangle \left|E_0\right\rangle \left|O_0\right\rangle \rightarrow \sum c_i \left|\varphi_i\right\rangle \left|A_i\right\rangle \left|E_i\right\rangle \left|O_i\right\rangle$$

The brain of the observer remains in a superposed state but her /his consciousness is hung up to only one branch which is chosen at random according to the Born rule

H. Zwirn, Les limites de la connaissance, Odile Jacob, 2000

The Decoherence Process

The Convivial Solipsism

The Hanging up mechanism:

Once the consciousness is hung up to one branch, it will hang up only to branches that are daughters of this branch for all the following observations.

That guarantees:

- That repeating the same measurement will give again the same result
- Any conflict with another observer is impossible

The Decoherence Process

The Convivial Solipsism

$\psi_s = [a|+>_z + b|->_z]$

$\psi^{f}_{SAE} = a|+>_z|\uparrow> |E+> + b|->_z |\downarrow> |E->$

$\psi_S\psi_A\,\psi_E\,\psi_O \rightarrow \psi^{f}_{SAEO} = a|+>_z|\uparrow> |E+> |☺> + b|->_z |\downarrow> |E->|☹>$

$C_O \rightarrow |☺>$ or $C_O \rightarrow |☹>$

The Decoherence Process

The Convivial Solipsism

Solipsism: Each observer lives inside his own branch indendently of the others

Convivial: No conflict is possible

You can think that you talk with a friend about your holidays while he thinks that he is swimming (but that doesn't mean much since there is no way to synchronize)

God doesn't play dice: you do!