High precision gravitational self-force calculations and post-Newtonian implications

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Outline

• Binary systems and gravitational waves
• SF-PN domain
• What is self force?
• Achieving high precision!
• Why do the comparison?
• Impact on gravitational waveforms?
• The advent of second order
Gravitational wave sources

- stochastic background - cosmic or astrophysical origin
- non-axisymmetric pulsars - with time-dependent quadrupole moment
- supernova explosions - not symmetric
- binary systems - inspiral, merger and ringdown
  - mass ratio 1-100: ground-based focus
  - mass ratio $10^3$-$10^9$: eLISA focus
  - compact EMRIs: our main focus
Inspiral and merger of a compact (NS/BH) binary system
A Global Network of Interferometers

LIGO Hanford 4 & 2 km

GEO Hannover 600 m

Kagra Japan 3 km

LIGO Livingston 4 km

Virgo Cascina 3 km

LIGO South Indigo
eLISA, would be a space-borne experiment to detect gravitational waves from EMRIs
Domains of interest

Post-Newtonian Theory

Perturbation Theory

Numerical Relativity

Mass Ratio

\[ \log_{10}(r/m) \]

\[ \log_{10}(m_2/m_1) \]
Numerical relativity best for equal masses; fails for binary systems with an extreme mass ratio.

innermost circular orbit

r = 6M
What is the self force problem?

- A small massive particle moves under the gravitational influence of a much larger mass
- At zeroth order the path is a geodesic
- At next order the path is altered
- There are both conservative (orbital) and dissipative (radiation reaction) effects occurring
- What is the new path of the particle?
- What is a computational scheme to find it?
Self-force causes deviation from background geodesic
Why is it a “problem”?

- In GR, a curve can be well defined, but the orbit of an extended body cannot be so easily defined.

- If one takes the singular limit, the geometry is not differentiable - required for the self force.

- Even if defined, the self-force is not gauge invariant; only the orbital phase at infinity is well defined.

- To calculate the orbital phase, one has to evolve the orbit, and one must choose a gauge in which to do it.
The rôle of the self-force

• At lowest order, point particles should follow geodesics
• Geodesic motion in a stationary space-time background does not radiate gravitational energy
• Extreme Mass Ratio Inspiraling (EMRI) Binary systems portray enormously different length scales and occur over long timescales (hundreds of thousands of orbits)
• Geodesic motion in a regularized space-time captures the inspiral evolution and demonstrate the radiation of gravitational energy
• Need to be able to follow the evolution AND include second order effects in order to enable sufficient precision for detection
Punctuated History

- 1938 Dirac
- 1960 De Witt
- 1997 MiSaTaQuWa
- Capra begins
- Regularization
- Singular field
- $L=0, 1$
- Time domain
- Gauge invariants
- Resonances
- Effective sources
- Scaling Limit
- Kerr black hole
- PN/EOB impact
- Second order
- Evolution
Physical situation

In the physical situation, there is no incoming radiation. Consequently, when the binary system loses energy due to the emission of gravitational waves, the orbit must evolve (shrink).
Helical Killing Vector situation

We work in a space-time with a helical Killing vector, in this case there is balancing incoming radiation and, in the computational domain, the orbit does not evolve (shrink).
Helical space-time symmetry

In the domain of each computation, the orbit does not actually evolve. Instead, there is a constant force on the particle, which the helical symmetry helps define and calculate simply. Note: $u^t$ is then gauge invariant.
Near and far zones overlap

Both self-force and post-Newtonian calculations require a near zone and a far zone. In both cases these must overlap. Matched asymptotic expansions are used to do the matching.
Matched asymptotic expansions

- Inner domain is described using multi-pole (far field) expansion around small source
- Outer domain uses perturbation around background of larger (rotating) mass or BH
- Expansions are matched in overlap region, relating inner and outer coefficients
- Equation of motion from simple multipole (body centered) choice for small object
internal and external zones; matching done in overlap region
Why do the post-Newtonian comparison?

- To show that we are on the right track
- To push the post-Newtonian expansion
- To contribute to pN Energy expression
- To contribute to Effective One Body (EOB) coefficients - pioneered in France

- Must work with a gauge invariants: $u^t$ and $\Omega$
Post-Newtonian approach

- General multipole expansion outside sources
- Start with vacuum solution at linear order
- Iterate to find expansion near $r=0$
- Create general solution inside sources
- Match asymptotically in overlap region
- Add “tails of tails” at higher pN order
How is SF high precision enabled?

- Solve a linear perturbation equation: \( \delta G_{\mu\nu} = \frac{8\pi G_N}{c^4} \delta T_{\mu\nu} \)
- Use circular orbits and frequency domain methods to deal with ordinary differential equations at a single frequency
- Represent solutions globally, heavily using machine algebra
- Use convergent sums of known (confluent and/or hypergeometric) functions with known overlap properties
- Calculate series parameter to sufficient precision in expansion parameter
- Regularize and mode sum numerically
The expansions

\[ R_H = e^{i\epsilon x} (-x)^{-2-i\epsilon} \sum_{n=-\infty}^{\infty} a_n F(n + \nu + 1 - i\epsilon, -n - \nu - i\epsilon, -1 - 2i\epsilon; x), \]
\[ R_\infty = e^{iz} z^{\nu-2} \sum_{n=-\infty}^{\infty} (-2z)^n b_n U(n + \nu + 3 - i\epsilon, 2n + 2\nu + 2; -2iz). \]

- where:
  \[ x = 1 - \frac{r}{2M}, \quad \epsilon = 2Mm\Omega \quad \text{and} \quad z = -\epsilon x \]

- and \( \nu \) is such that series converge at \( \pm \infty \)
What has been achieved?

• Initially?
  • confirmed coefficients up to 3PN, Ln*4/5pN
• More recently?
  • Detected coefficients at n.5pN and higher
  • Have analytic coefficients up to: 4pN, 8.5pN†, Ln*6pN, Ln*9.5pN, Ln²*9pN, Ln³*10pN
  • Have numerical coefficients up to 10-10.5pN
  • Have fluxes to 20pN order (Shah, 2014)
  • †Confirmed by pN up to 7.5pN order
Impact on gravitational waveforms?

• $u^t$ expanded in $\eta = \frac{Mm}{(M+m)^2} \leq 1/4$

• $E \rightarrow E^0 + \eta E^1$ effective throughout range

• EOB $a(u)$ directly related to SF result

• Controls matching with NR waveforms

• Higher orders do not tend to convergence

• For LISA, will require long time evolution
The advent of second order

\[ \tilde{U}^t = \frac{1}{\sqrt{1 - 3(M\Omega)^{2/3}}} \left\{ 1 + \frac{1}{2} \epsilon h_{u_0 u_0}^R \right. \\
+ \epsilon^2 \left[ \frac{1}{2} h_{u_0 u_0}^{R2} + \frac{3}{8} (h_{u_0 u_0}^R)^2 - \frac{1}{6\Omega^2} (1 - 3(M\Omega)^{2/3})(\hat{F}_{1r})^2 \right] + O(\epsilon^3) \right\} \]

- Adam Pound (1404.1543)
- pN already has an expression beyond 3pN
- Are these defined under the same conditions? Comparison will tell!
Summary

- Post-Newtonian analysis, perturbative calculations and full non-linear numerical relativity all play a rôle in exploring gravitational waveforms for binary sources.
- EMRIs have very different local timescales and require 100,000+ orbits.
- Post-Newtonian can calculate to all orders in m/M but require v/c small.
- Perturbative self-force calculations manage the highly relativistic regime; each order in m/M is separate.
- Both methods have singularities and treat radiative effects separately.
- Self-force calculations have extended the post-Newtonian expansion.
- Post-Newtonian will help identify the right comparison at 2nd SF order.