Atomism and Relationalism as guiding principles for Quantum Gravity

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“Frontiers of Fundamental Physics” (FFP14)
Marseille July 16th, 2013
CONTENT OF THE TALK

RELATIONALISM

- ONTOLOGY: Structural Spacetime Realism
- Relational QUANTUM MECHANICS (Rovelli 1996)
- Partial Observables in GENERAL RELATIVITY (Rovelli 2001)

ATOMISM

- QUANTUM MECHANICS is about discrete quanta!
- QUANTUM MECHANICS is about finite information!
- Quantum Gravity should extend GENERAL RELATIVITY in a discrete framework!
ONTIOLOGY

Substantival & Relational
ONTHOLOGY ACCORDING TO PHYSICS

- Newton: Particles \rightarrow Space \rightarrow Time
- Faraday-Maxwell: Particles \rightarrow Fields \rightarrow Space \rightarrow Time
- Special Relativity: Particles \rightarrow Fields \rightarrow Spacetime
- Quantum Mechanics: Quantum-Fields \rightarrow Spacetime
- General Relativity: General-covariant fields
- Quantum Gravity: General-covariant quantum fields
ATOMISM 1

Quantum Mechanics
Discreteness is the defining property of \( \text{QM} \).

Discreteness scale is given by \( \hbar \): an action, or phase-space volume.

\[ (\mathcal{F}, \mathcal{A}, W) \]

\[ F \ni |p_1 \ldots p_n\rangle \]

\[ \mathcal{A} \ni a(k), a^\dagger (k) \]

\( W \rightarrow \text{Feynman rules} \)
ATOMISM 2

the Planck length
QUANTUM MECHANICS
Heisenberg Uncertainty
Sharp localization requires large energy.

GENERAL RELATIVITY
Black-Hole Horizon
The horizon prevents a sharper localization.

QUANTUM GRAVITY

$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m$

“Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain.”

Matvei Bronstein
RELATIONALISM 1

Quantum Mechanics
Almost all the interpretations of quantum mechanics

**two systems**: the observed system and the system that observes

for instance:

- **Copenhagen**: there is always the measurement apparatus
- **Many-world**: what is observed is not the absolute value of a quantity but the value in the "branch" where is the observer.

**RELATIONAL QUANTUM MECHANICS** moves from this fact:  

the values of variables in the MQ are *always* relational  

**MQ**: distinct observers may give different accounts of the same sequence of events

**States**:
- All quantum states are relative states  
  [à la Everett]
- States refer to systems in relation to other systems  
  [à la Galilei]
- quantum state = way of coding the result of past interactions

knowledge of past interactions → predictions about outcome of future interactions
**Localization**: not given with respect to a fixed background structure. Dynamical object localized with respect to one another [Rovelli 1990]

**Partial Observables**: are not predictable individually, but that can be measured. Knowledge of some of them allows us to predict the others [Rovelli 2002]

**Example**: in the Lorentz-invariant description of a relativistic particle, all coordinates are partial observable $x^\mu$. All we can predict are the relationship between them.

The relevant relation that builds the spacetime structure is **contiguity**: the fact of being “next to one another” in spacetime. A general relativistic theory = a dynamical patchwork of adjacent spacetime regions.
Interactions are local \(\leftrightarrow\) objects are contiguous if they interact.

A process is not in a spacetime region: a process is a spacetime region.

Boundary between processes can be moved at wish.
Final total amplitudes are not affected by displacing the boundary between “observed system” and “observing system”.

Boundaries are arbitrarily drawn in spacetime.
Partitions are at the same time subsystems split and partitions of spacetime.

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**LOCALITY**

**QUANTUM MECHANICS**

Process \(\leftrightarrow\) Locality
State

**GENERAL RELATIVITY**

Spacetime region
Boundary, space region
Spacetime is a process, a state is what happens at its boundary.

Boundary state \( \Psi = \psi_{in} \otimes \psi_{out} \)

Amplitude of the process \( A = W(\Psi) \)

LOOP QUANTUM GRAVITY gives a mathematical definition of the state of space, the boundary observables, and the amplitude of the process.
And God said

(\mathcal{H}, \mathcal{A}, \mathcal{W})

and there was SpaceTime

Hilbert Space:
\mathcal{H}_\Gamma = L_2[\text{SU}(2)^L/\text{SU}(2)^N]

Transition Amplitude:
W_\nu = (P_{\text{SL}(2,\mathbb{C})} \circ Y_\gamma \psi_\nu)(\mathbf{I})

Operator Algebra:
[L^i_a, L^j_b] = i\delta_{ab}l^2\epsilon^{ij}k \ L^k_a
ATOMISM & RELATIONALIMS

Quanta of Spacetime
Loop Quantum Gravity is a theory about quanta of spacetime
Quanta have a locally Lorentz covariant description
The states are boundary states at fixed time
The physical phase space is spanned by SU(2) group variables
Abstract graphs: \( \Gamma = \{N, L\} \)

Group variables:
\[
\begin{cases}
    h_l \in SU(2) \\
    \bar{L}_l \in su(2)
\end{cases}
\]

Graph Hilbert space: \( \mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N] \)

The space \( \mathcal{H}_\Gamma \) admits a basis \( |\Gamma, j_l, \nu_n\rangle \)

Gauge invariant operator \( G_{ll'} = \bar{L}_l \cdot \bar{L}_{l'} \) with \( \sum_{l \in n} G_{ll'} = 0 \)

Penrose’s spin-geometry theorem (1971), and Minkowski theorem (1897)

\( h_l \) “Holonomy of the Ashtekar-Barbero connection along the link”

\( \bar{L}_l = \{L^i_l\}, i = 1, 2, 3 \) SU(2) generators
\( L^i \psi(h) = \frac{d}{dt} \psi(h e^{t \tau_i}) \bigg|_{t=0} \)

gravitational field operator (tetrad)
Composite operators:

- **Area:** \( A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i} \).
- **Volume:** \( V_R = \sum_{n \in R} V_n, \quad V_n^2 = \frac{2}{9} |\epsilon_{ijk} L_l^i L_l^j L_l^k| \).
- **Angle:** \( L_l^i L_l^i \).

Geometry is quantized:
- eigenvalues are discrete
- the operators do not commute
- quantum superposition
  - coherent states

**Quantum states of space, rather than states on space.**
Probability amplitude \( P(\psi) = |\langle W | \psi \rangle|^2 \)

Amplitude associated to a state \( \psi \) of a **boundary** of a 4d region

- **Superposition principle** \( \langle W | \psi \rangle = \sum_\sigma W(\sigma) \)
- **Locality:** vertex amplitude \( W(\sigma) \sim \prod_v W_v \).
- **Lorentz covariance** \( W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \psi_v)(1) \)

**Classical limit:** GR

Barrett, Dowall, Fairbain, Gomes, Hellmann, Alesci...’09
Coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry.

The kinematics and the dynamics are the one of the full quantum theory.

The kinematics provide minimal eigenvalue for geometrical quantities. The dynamics provide a bound on the curvature and on the acceleration. This provide a mechanism to remove GR singularities. [Rovelli, FV 2013]

The amplitudes are peaked on the semiclassical solutions. Verified for FLWD and deSitter.

New framework for the study of primordial cosmological fluctuations. It is an approximated kinematics of the universe, inhomogeneous but truncated to a finite number of cells. [Bianchi, Rovelli, FV 2010]
RELATIONAL INTERPRETATION OF QUANTUM MECHANICS
quantum-mechanical variables are always relational

PARTIAL OBSERVABLES: THERE IS NO ABSOLUTE TIME
but many relational times governed by interactions

QUANTA OF SPACE
prediction of Loop Quantum Gravity
fundamental scale: the Planck length

“Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain.”

Matvei Bronstein
to conclude:
What does exist? Does space exists? or it emerges from the relations between bodies?

Quantum Gravity is the quest for a synthesis between Quantum Mechanics and General Relativity. But while doing this, quantum gravity would achieve a synthesis also between substantivalism and relationalism:

spacetime is a field, that comes to existence only trough its interactions.

Atomism is fundamental: everything is made of discrete quanta. The quantum discreteness of spacetime come from its Lorentz symmetry.

Relationalism is fundamental: both in Quantum Mechanics and General Relativity.

Space is constitute of atoms of space, defined trough their relations.

SPACETIME IS A PROCESS

Loop Quantum Gravity is a concrete realizations of these ideas.