Group field theories generating polyhedral complexes

Johannes Thüриgen
w.i.p. with D. Oriti, J. Ryan

Max-Planck Institute for Gravitational Physics, Potsdam
(Albert-Einstein Institute)

July 17, 2014
Dynamics of LQG: Group field theory (GFT)

Standard GFT: QFT of a field on a group $\phi : G \times D \longrightarrow \mathbb{R}$

$$S[\phi] = \int [dg] \phi(g_1) \mathcal{K}(g_1, g_2) \phi(g_2) + \sum_{i \in I} \lambda_i \int [dg] \mathcal{V}_i(\{g_e\}_{E_i}) \prod_{e \in E_i} \phi(g_e),$$

Perturbative expansion of state sum = sum over spin foams

$$Z_{GFT} = \int \mathcal{D}\phi \ e^{-S[\phi]} = \sum_{\Gamma} \frac{1}{C(\Gamma)} \left[ \prod_{i \in I} (-\lambda_i)^{V_i} \right] A(\Gamma)$$
Apparent difference between canonical and covariant approaches to LQG:

- States in canonical LQG labeled by graphs of arbitrary valence
- Covariant theories developed in simplicial setting
- Generalization of spin foams to arbitrary valence exists [KKL '10]
- Slightly more challenging for GFT
GFT for arbitrary graphs?

Apparent difference between canonical and covariant approaches to LQG:

- States in canonical LQG labeled by graphs of arbitrary valence
- Covariant theories developed in simplicial setting
- Generalization of spin foams to arbitrary valence exists [KKL '10]
- Slightly more challenging for GFT

Goal here:

- Define the general class of combinatorial complexes relevant for LQG and Spin Foams
- Discuss equivalent diagrammatic representations for them
- Present two types of GFTs generating those in the perturbative sum
GFT for arbitrary graphs?

Apparent difference between canonical and covariant approaches to LQG:

- States in canonical LQG labeled by graphs of arbitrary valence
- Covariant theories developed in simplicial setting
- Generalization of spin foams to arbitrary valence exists \([KKL \ '10]\)
- Slightly more challenging for GFT

Goal here:

- Define the general class of *combinatorial* complexes relevant for LQG and Spin Foams
- Discuss equivalent diagrammatic representations for them
- Present two types of GFTs generating those in the perturbative sum

Caveat: Focus here on (dual) polyhedral 2-complexes (just polygons)
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte ’02]
Mathematical background: Abstract polyhedral complexes

"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte '02]

(P1) $P$ contains a least and greatest face $f_{-1}, f_n$. 
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ \cite{McMullen,Schulte '02}

(P1) $P$ contains a least and greatest face $f_{-1}, f_n$.

(P2) Each maximal chain has length $n + 1$
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte ’02]

(P1) $P$ contains a least and greatest face $f_{-1}, f_n$.
(P2) Each maximal chain has length $n + 1$
(P3) $P$ is strongly connected (sequence of faces)
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte '02]

(P1) $P$ contains a least and greatest face $f_{-1}$, $f_n$.
(P2) Each maximal chain has length $n + 1$
(P3) $P$ is strongly connected (sequence of faces)
(P4) Homogeneity degrees $k_i = 2$, $i = 1, .., n - 1$
   (for $f < g$ of dimension $i-1$ and $i+1$ there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)
Mathematical background: Abstract polyhedral complexes

"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte ’02]

(P1) $P$ contains a least and greatest face $f_{-1}$, $f_n$.
(P2) Each maximal chain has length $n + 1$
(P3) $P$ is strongly connected (sequence of faces)
(P4) Homogeneity degrees $k_i = 2$, $i = 1, .., n - 1$
  (for $f < g$ of dimension $i-1$ and $i+1$ there are exactly
  $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte '02]

(P1) $P$ contains a least and greatest face $f_{-1}$, $f_n$.  
(P2) Each maximal chain has length $n + 1$  
(P3) $P$ is strongly connected (sequence of faces)  
(P4) Homogeneity degrees $k_i = 2$, $i = 1, .., n − 1$  
(for $f < g$ of dimension $i$-1 and $i+1$ there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)
Mathematical background: Abstract polyhedral complexes

"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte '02]

(P1) $P$ contains a least and greatest face $f_{-1}$, $f_n$. 
(P2) Each maximal chain has length $n + 1$
(P3) $P$ is strongly connected (sequence of faces)
(P4) Homogeneity degrees $k_i = 2$, $i = 1, .., n - 1$
   (for $f < g$ of dimension $i$-1 and $i$+1 there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)

- more general than piecewise linear (e.g. 11-cell)
Mathematical background: Abstract polyhedral complexes

"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte '02]

(P1) $P$ contains a least and greatest face $f_{-1}, f_n$.

(P2) Each maximal chain has length $n + 1$

(P3) $P$ is strongly connected (sequence of faces)

(P4) Homogeneity degrees $k_i = 2, i = 1, .., n - 1$

(for $f < g$ of dimension $i-1$ and $i+1$ there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)

- more general than piecewise linear (e.g. 11-cell)
- dual polytope: invert partial order
"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ \cite{McMullen1980}

(P1) $P$ contains a least and greatest face $f_{-1}$, $f_n$.

(P2) Each maximal chain has length $n + 1$

(P3) $P$ is strongly connected (sequence of faces)

(P4) Homogeneity degrees $k_i = 2$, $i = 1, \ldots, n - 1$
  
  (for $f < g$ of dimension $i-1$ and $i+1$ there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)

- more general than piecewise linear (e.g. 11-cell)
- dual polytope: invert partial order
Mathematical background: **Abstract polyhedral complexes**

"Combinatorial complexes = complexes of abstract polytopes"

An abstract $n$-polytope is a poset $P$ [McMullen, Schulte ’02]

(P1) $P$ contains a least and greatest face $f_{-1}, f_n$.

(P2) Each maximal chain has length $n + 1$

(P3) $P$ is strongly connected (sequence of faces)

(P4) Homogeneity degrees $k_i = 2, i = 1, .., n - 1$

(for $f < g$ of dimension $i-1$ and $i+1$ there are exactly $k_i$ $i$-faces $h$ in $P$ such that $f < h < g$)

- more general than piecewise linear (e.g. 11-cell)
- dual polytope: invert partial order
- boundary is a closed manifold $\rightarrow$ generalize!
Mathematical background: Abstract polyhedral complexes

"Combinatorial complexes = complexes of abstract polytopes"

An abstract polyhedral $n$-complex is a poset $P$

(P1) $P$ contains least and greatest face $f_{-1}$, $f_{n+1}$.

(P2) Each maximal chain has length $n + 2$

(P3) ($P$ is strongly connected)

(P4') Homogeneity degrees $k_i = 2, i = 1, .., n - 1$

- more general than piecewise linear (e.g. 11-cell)
- dual polytope branching possible!
Mathematical background: Abstract polyhedral complexes

”Combinatorial complexes = complexes of abstract polytopes”

A generalized abstract polyhedral $n$-complex is a poset

(P1) $P$ contains least and greatest face $f_{-1}, f_{n+1}$.
(P2) Each maximal chain has length $n + 2$
(P3) ($P$ is strongly connected)
(P4’’) Face degree $k_i^f \in \{1, 2\}$, $i = 1, ..., n - 1$

- more general than piecewise linear (e.g. 11-cell)
- dual polytope branching possible!
Representations of Polyhedral 2-complexes

LQG/SF: 2-complexes, boundary 1-complexes
- finite abstract 2-polytopes are polygons
- alternative representation: stranded diagrams (faces represented by strands)
Representations of Polyhedral 2-complexes

LQG/SF: 2-complexes, boundary 1-complexes
- finite abstract 2-polytopes are polygons
- alternative representation: stranded diagrams (faces represented by strands)
Representations of Polyhedral 2-complexes

LQG/SF: 2-complexes, boundary 1-complexes

- finite abstract 2-polytopes are polygons
- alternative representation: stranded diagrams (faces represented by strands)
- subdivision $\rightarrow$ edges/faces represented by vertices
LQG/SF: 2-complexes, boundary 1-complexes

- finite abstract 2-polytopes are polygons
- alternative representation: stranded diagrams (faces represented by strands)
- subdivision → edges/faces represented by vertices
- subdivided cells adjacent to bulk vertex define spin foam atoms
Atomic decomposition

A spin foam atom consists of

- a bulk vertex \( v \)
- boundary vertices \( \overline{V} = \{ \overline{v}_i, \ldots \} \), isomorphic to bulk edges \( (v \overline{v}_i) \)
- bisection vertices \( \hat{V} = \{ \hat{v}_{ij}, \ldots \} \), isomorphic to face wedges \( (v \overline{v}_i \hat{v}_{ij} \overline{v}_j) \)
- boundary (half)edges \( (\overline{v}_i \hat{v}_{ij}), (\overline{v}_j \hat{v}_{ij}) \)
Atomic decomposition

A spin foam atom consists of

- a bulk vertex $\nu$
- boundary vertices $\overline{V} = \{\overline{v}_i, \ldots\}$, isomorphic to bulk edges $(\nu \overline{v}_i)$
- bisection vertices $\hat{V} = \{\hat{v}_{ij}, \ldots\}$, isomorphic to face wedges $(\nu \overline{v}_i \hat{v}_{ij} \overline{v}_j)$
- boundary (half)edges $(\overline{v}_i \hat{v}_{ij}), (\overline{v}_j \hat{v}_{ij})$

interpretation as dual to (local) $D$-polytope possible, but not unique
Atomic decomposition

A spin foam atom consists of

- a bulk vertex \( v \)
- boundary vertices \( \overline{V} = \{ \overline{v}_i, \ldots \} \), isomorphic to bulk edges \( (v \overline{v}_i) \)
- bisection vertices \( \hat{V} = \{ \hat{v}_{ij}, \ldots \} \), isomorphic to face wedges \( (v \overline{v}_i \hat{v}_{ij} \overline{v}_j) \)
- boundary (half)edges \( (\overline{v}_i \hat{v}_{ij}), (\overline{v}_j \hat{v}_{ij}) \)

interpretation as dual to (local) \( D \)-polytope possible, but not unique

Spin foam atoms are isomorphic to bisected closed graphs (cf. [KKL '10, KLP '12])

(which are isomorphic to closed graphs)
Foams as molecules from atoms

- Spin foams are bondings of atoms along patches

\[ \text{GFT: Wick contractions of fields } \phi(g \bar{v} \hat{v}_1, \ldots, g \bar{v} \hat{v}_n) \]

Boundary given by patches which are not bonded by deletion of internal (closed) strands
Foams as molecules from atoms

Spin foams are bondings of atoms along patches

GFT: Wick contractions of fields

\[ \phi(g\bar{v}v_1, \ldots, g\bar{v}v_n) \]
Foams as molecules from atoms

- Spin foams are bondings of atoms along patches
- GFT: Wick contractions of fields
  \[ \phi(g_{\bar{v}, v_1}, \ldots, g_{\bar{v}, v_n}) \]
Foams as molecules from atoms

- Spin foams are bondings of atoms along patches
- GFT: Wick contractions of fields $\phi(g\bar{v}\hat{v}_1, \ldots, g\bar{v}\hat{v}_n)$
- Boundary given by patches which are not bonded/ by deletion of internal (closed) strands
Foams as molecules from atoms

- Spin foams are bondings of atoms along patches
- GFT: Wick contractions of fields 
  \[ \phi(g_{\bar{v}v_1}, \ldots, g_{\bar{v}v_n}) \]
- Boundary given by patches which are not bonded by deletion of internal (closed) strands

Any generalized polyhedral (2-)complex has a decomposition into atoms, and equivalently is a bonding of atoms
Foams as molecules from atoms

- Spin foams are bondings of atoms along patches
- GFT: Wick contractions of fields
  \( \phi(g_{\vec{v}_1}, \ldots, g_{\vec{v}_n}) \)
- Boundary given by patches which are not bonded by deletion of internal (closed) strands

Any generalized polyhedral (2-)complex has a decomposition into atoms, and equivalently is a bonding of atoms

All of this works for generalized polyhedral complexes of arbitrary dimension
Generalizing the combinatorics of simplicial GFT

- No obstacle for GFT of polyhedral interactions with simplicial boundary
Generalizing the combinatorics of simplicial GFT

- No obstacle for GFT of polyhedral interactions with simplicial boundary
- Important: criteria, analytical/numerical control
Generalizing the combinatorics of simplicial GFT

- No obstacle for GFT of polyhedral interactions with simplicial boundary
- Important: criteria, analytical/numerical control
- Polytopes effectively generated anyway:
  Ex.: Uncoloring of colored GFT (bipartite simplex gluings) [Bonzom, Gurau, Rivasseau '12]
Generalizing the combinatorics of simplicial GFT

- No obstacle for GFT of polyhedral interactions with simplicial boundary
- Important: criteria, analytical/numerical control
- Polytopes effectively generated anyway: Ex.: Uncoloring of colored GFT (bipartite simplex gluings) [Bonzom, Gurau, Rivasseau ’12]
- Decomposition generalizes to GFT without colors: Any regular boundary atoms generated by simplicial interaction
Generalizing the combinatorics of simplicial GFT

- No obstacle for GFT of polyhedral interactions with simplicial boundary
- Important: criteria, analytical/numerical control
- Polytopes effectively generated anyway: Ex.: Uncoloring of colored GFT (bipartite simplex gluings) [Bonzom, Gurau, Rivasseau '12]
- Decomposition generalizes to GFT without colors: Any regular boundary atoms generated by simplicial interaction

Prop.: Any atom with regular boundary graph can be decomposed into simplicial atoms
Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli ‘01]

Extend field content $\phi(l) : G^\times I \to \mathbb{R}$
Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli '01]

Extend field content $\phi(l) : G^{\times l} \rightarrow \mathbb{R}$

$$S[\{\phi(l)\}] = \sum_l \int [dg] \phi(l)(g_1) K(l)(g_1, g_2) \phi(l)(g_2) + \sum_{i \in I} \lambda_i \int [dg] \nu_i(\{g \bar{v}\} \bar{V}_i) \prod_{\bar{v} \in \bar{V}_i} \phi(l)(g \bar{v})$$

...
Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli '01]

Extend field content \( \phi(l) : G \times l \rightarrow \mathbb{R} \)

\[
S[\{\phi(l)\}] = \sum_l \int [dg] \phi(l)(g_1) K(l)(g_1, g_2) \phi(l)(g_2) + \sum_{i \in I} \lambda_i \int [dg] \mathcal{V}_i (\{g_{\bar{v}}\}_{\bar{V}_i}) \prod_{\bar{v} \in \bar{V}_i} \phi(l)(g_{\bar{v}})
\]

• extension to \( D \)-polytopes not specified
Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli '01]

Extend field content $\phi(l) : G \times l \rightarrow \mathbb{R}$

$$S[\{\phi(l)\}] = \sum_l \int [dg] \phi(l)(g_1) K(l)(g_1, g_2) \phi(l)(g_2) + \sum_{i \in I} \lambda_i \int [dg] V_i (\{g_{\bar{v}}\}_{\bar{V}_i}) \prod_{\bar{v} \in \bar{V}_i} \phi(l)(g_{\bar{v}})$$

- extension to $D$-polytopes not specified
- infinite field species $l \in \mathbb{N}$ formally possible
Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli ’01]

Extend field content $\phi(l) : G^\times l \longrightarrow \mathbb{R}$

$$S[\{\phi(l)\}] = \sum_l \int [dg] \phi(l)(g_1) \mathcal{K}(l)(g_1, g_2) \phi(l)(g_2) + \sum_{i \in I} \lambda_i \int [dg] \nu_i(\{g_{\bar{v}}\}_{\bar{v}_i}) \prod_{\bar{v} \in \bar{V}_i} \phi(l)(g_{\bar{v}})$$

- extension to $D$-polytopes not specified
- infinite field species $l \in \mathbb{N}$ formally possible
- GFT generating known Spin Foams [KKL ’10]
Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli ’01]

Extend field content \( \phi(l) : G^l \rightarrow \mathbb{R} \)

\[
S[\{\phi(l)\}] = \sum_l \int [dg] \phi(l)(g_1) K_l(g_1, g_2) \phi(l)(g_2) + \sum_{i \in I} \lambda_i \int [dg] V_i(\{g_{\bar{v}}\}_{V_i}) \prod_{\bar{v} \in \bar{V}_i} \phi(l)(g_{\bar{v}})
\]

- extension to \( D \)-polytopes not specified
- infinite field species \( l \in \mathbb{N} \) formally possible
- GFT generating known Spin Foams [KKL ’10]
- practical usefulness?
Virtual edges

Correspondence to regular graphs

- $k$ odd: Any graph can be obtained from a $k$-regular graph by contraction/deletion of edges labeled as virtual
- $k$ even: Any graph with even valencies

Advantage: Only patches with fixed valency needed!
Dually weighted GFT

Fields with indices $m_{\tilde{v}\tilde{v}} = 0, 1, \ldots, M$: $\phi_{m_{\tilde{v}}} (g_{\tilde{v}}) = \phi_{m_{\tilde{v}1}, \ldots, m_{\tilde{v}D}} (g_{\tilde{v}1}, \ldots, g_{\tilde{v}D})$

Meaning: edge physical $m = 0$, virtual $m > 1$
Dually weighted GFT

Fields with indices $m_{\bar{v}} = 0, 1, \ldots, M$: $\phi_{m_{\bar{v}}}(g_{\bar{v}}) = \phi_{m_{\bar{v}1}, \ldots, m_{\bar{v}D}}(g_{\bar{v}1}, \ldots, g_{\bar{v}D})$

Meaning: edge physical $m = 0$, virtual $m > 1$

$$
S[\phi] = \sum_{m_1, m_2} \int [dg] \phi_{m_1}(g_1) K(g_1, g_2) K_{m_1, m_2}^{(M)} \phi_{m_2}(g_2) \\
+ \sum_{i \in I} \lambda_i \sum_{\{m_{\bar{v}}\}_{\bar{v}}} \int [dg] \mathcal{V}_i(\{g_{\bar{v}}\}_{\bar{v}i}) \mathcal{V}_{\{m_{\bar{v}}\}_{\bar{v}}}^{(M)} \prod_{\bar{v} \in \bar{v}_i} \phi(g_{\bar{v}})
$$
Dually weighted GFT

Fields with indices \( m_{\bar{v}} = 0, 1, \ldots, M \): \( \phi_{m_{\bar{v}}}(g_{\bar{v}}) = \phi_{m_{\bar{v}}_{1}, \ldots, m_{\bar{v}}_{D}}(g_{\bar{v}}_{1}, \ldots, g_{\bar{v}}_{D}) \)

Meaning: edge physical \( m = 0 \), virtual \( m > 1 \)

\[
S[\phi] = \sum_{m_{1}, m_{2}} \int [dg] \phi_{m_{1}}(g_{1}) \mathcal{K}(g_{1}, g_{2}) \mathcal{K}_{m_{1}, m_{2}}^{(M)} \phi_{m_{2}}(g_{2})
\]

\[
+ \sum_{i \in I} \lambda_{i} \sum_{\{m_{\bar{v}}\}_{\bar{V}}} \int [dg] \mathcal{V}_{i}(\{g_{\bar{v}}\}_{\bar{V}_{i}}) \mathcal{V}_{\{m_{\bar{v}}\}_{\bar{V}}}^{(M)} \prod_{\bar{v} \in \bar{V}_{i}} \phi(g_{\bar{v}})
\]

- using a matrix \( A \), \( \lim_{M \to \infty} \frac{1}{M} \text{Tr} A^{q} = \delta_{q,2} \):

\[
\mathcal{K}_{m_{1}m_{2}}^{(M)} = \prod_{i=1}^{D} \mathcal{K}_{m_{1}, m_{2}, i}^{(M)} = \begin{pmatrix} 1 & 0 \\ 0 & A \end{pmatrix}^{m_{1}, i m_{2}, i}
\]
Dually weighted GFT

Fields with indices $m_{\bar{v}\bar{v}} = 0, 1, \ldots, M$: $\phi_{m_{\bar{v}}} (g_{\bar{v}}) = \phi_{m_{\bar{v}1}, \ldots, m_{\bar{v}D}} (g_{\bar{v}1}, \ldots, g_{\bar{v}D})$

Meaning: edge physical $m = 0$, virtual $m > 1$

$$S[\phi] = \sum_{m_1, m_2} \int [dg] \phi_{m_1} (g_1) K(g_1, g_2) K^{(M)}_{m_1, m_2} \phi_{m_2} (g_2)$$

$$+ \sum_{i \in I} \lambda_i \sum_{\{m_{\bar{v}}\}_{\bar{V}}} \int [dg] \nu_i (\{g_{\bar{v}}\}_{\bar{V}_i}) \nu^{(M)}_{\{m_{\bar{v}}\}_{\bar{V}}} \prod_{\bar{v} \in \bar{V}_i} \phi (g_{\bar{v}})$$

- using a matrix $A$, $\lim_{M \to \infty} \frac{1}{M} Tr A^q = \delta_{q,2}$:
  $$K^{(M)}_{m_1 m_2} = \prod_{i=1}^{D} K^{(M)}_{m_1, i m_2, i} = \begin{pmatrix} 1 & 0 \\ 0 & A \end{pmatrix}^{m_1, i m_2, i}$$

- All physical quantities in the limit $\lim_{M \to \infty}$
Dually weighted GFT

Fields with indices \( m_{\bar{v}} = 0, 1, \ldots, M \): \( \phi_{m_{\bar{v}}}(g_{\bar{v}}) = \phi_{m_{\bar{v}1}, \ldots, m_{\bar{v}D}}(g_{\bar{v}1}, \ldots, g_{\bar{v}D}) \)

Meaning: edge physical \( m = 0 \), virtual \( m > 1 \)

\[
S[\phi] = \sum_{m_1, m_2} \int [dg] \phi_{m_1}(g_1) K(g_1, g_2) K_{m_1, m_2}^{(M)} \phi_{m_2}(g_2)
\]

\[
+ \sum_{i \in I} \lambda_i \sum_{\{m_{\bar{v}}\}_{\bar{v}}} \int [dg] V_i(\{g_{\bar{v}}\}_{\bar{v}i}) V_{\{m_{\bar{v}}\}_{\bar{v}}}^{(M)} \prod_{\bar{v} \in \bar{v}_i} \phi(g_{\bar{v}})
\]

- using a matrix \( A \), \( \lim_{M \to \infty} \frac{1}{M} Tr A^q = \delta_{q,2} \):
  \[
  K_{m_1 m_2}^{(M)} = \prod_{i=1}^{D} K_{m_1, i, m_2, i}^{(M)} = \begin{pmatrix} 1 & 0 \\ 0 & A \end{pmatrix}_{m_1, i, m_2, i}
  \]

- All physical quantities in the limit \( \lim_{M \to \infty} \)

Result: Virtual edges are not dynamical, propagation as composite objects
Gravitational models, dually weighted

No obstacle for including the established amplitudes

Ex.: **EPRL edge operator** on edge $\bar{v}$:

$$\mathcal{K}(g_{v_1\bar{v}}, g_{v_2\bar{v}}) = \int_{G^2} dh_{v_1\bar{v}} dh_{v_2\bar{v}} \prod_{(\bar{v}\bar{\imath})} \sum_{J_{\bar{v}} \in \mathcal{J}} \text{tr}_{J_{\bar{v}}} (g_{v_1\bar{v}\bar{\imath}} h_{v_1\bar{v}}^{-1} S_{J_{\bar{v}}, N_{\bar{v}}} h_{v_2\bar{v}} g_{v_2\bar{v}\bar{\imath}}^{-1})$$

$(S_{J,N}$ simplicity operator, $\mathcal{J}$ set of $\gamma$–simple reps of $g = so(4) \cong su(2)_+ \times su(2)_-$)

- imposes simplicity on "triangulated" atom
- alternative: trivial factor $\delta(g_{v_1\bar{v}\bar{\imath}} h_{v_1\bar{v}}^{-1}) \delta(h_{v_2\bar{v}} g_{v_2\bar{v}\bar{\imath}}^{-1})$ on virtual links
- modification of geometricity for higher valency?
Conclusions

- combinatorial compatibility of any LQG and GFT shown
- higher-valent GFT models: multi-species, dual weighting
- precise understanding of generated complexes
Conclusions

- combinatorial compatibility of any LQG and GFT shown
- higher-valent GFT models: multi-species, dual weighting
- precise understanding of generated complexes

Stage is set to analyze these models

- effective interactions, relevant operators
- ...

(though from a QFT perspective the generalization seems rather unnecessary and complicated)