Group field theories generating polyhedral complexes

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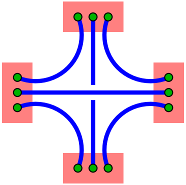
Dynamics of LQG: Group field theory (GFT)

Standard GFT: QFT of a field on a group $\phi : G^{\times D} \longrightarrow \mathbb{R}$

$$S[\phi] = \int [\mathrm{d}g]\, \phi(g_1)\, \mathcal{K}(g_1, g_2)\, \phi(g_2) + \sum_{i\in I} \lambda_i \int [\mathrm{d}g]\, \mathcal{V}_i(\{g_e\}_{E_i}) \prod_{e\in E_i} \phi(g_e)\,,$$



Perturbative expansion of state sum = sum over spin foams

$$Z_{GFT} = \int \mathcal{D}\phi\; e^{-S[\phi]} = \sum_{\Gamma} \frac{1}{C(\Gamma)} \Big[\prod_{i\in I} (-\lambda_i)^{V_i} \Big] A(\Gamma)$$

GFT for arbitrary graphs?

Apparent difference between canonical and covariant approaches to LQG:

- States in canonical LQG labeled by graphs of arbitrary valence
- Covariant theories developed in simplicial setting
- Generalization of spin foams to arbitrary valence exists [KKL '10]
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Goal here:

- Define the general class of *combinatorial* complexes relevant for LQG and Spin Foams
- Discuss equivalent diagrammatic representations for them
- Present two types of GFTs generating those in the perturbative sum

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Caveat: Focus here on (dual) polyhedral 2-complexes (just polygons)

"Combinatorial complexes = complexes of abstract polytopes"

An abstract n-polytope is a poset P [McMullen,Schulte '02]

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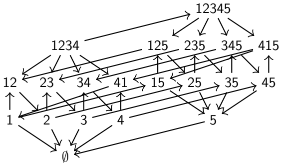

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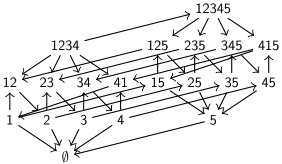

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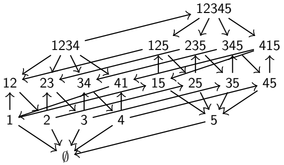

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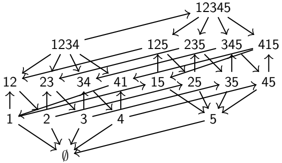

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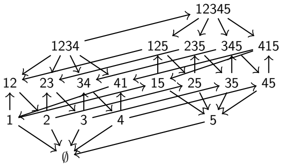

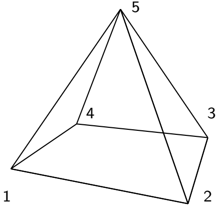

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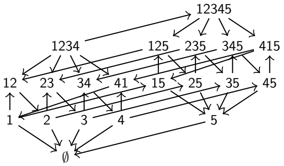

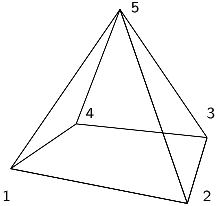

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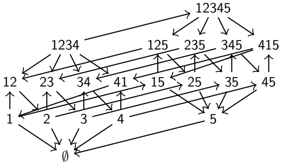

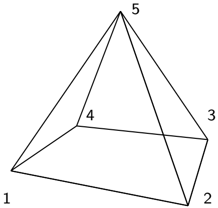

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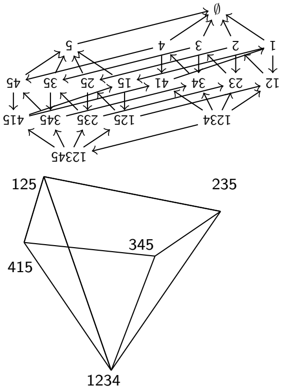

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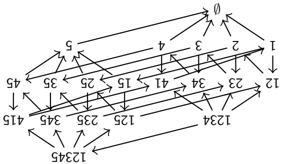

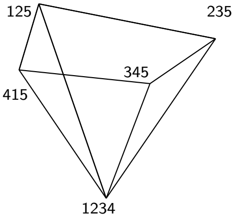

- more general than piecewise linear (e.g. 11-cell)
- dual polytope: invert partial order
- boundary is a closed manifold → generalize!

"Combinatorial complexes = complexes of abstract polytopes"

An abstract polyhedral n-complex is a poset P

(P1) P contains least and greatest face f_{-1}, f_{n+1}.

(P2) Each maximal chain has length $n+2$

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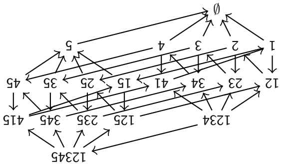
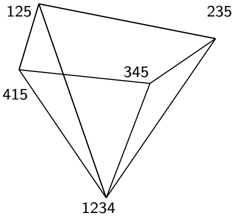
- ~~dual polytope~~ branching possible!





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(P4") Face degree $k_i^f \in \{1,2\}$, $i = 1,..,n-1$

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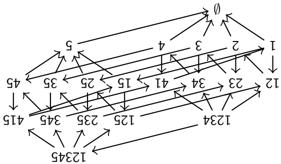
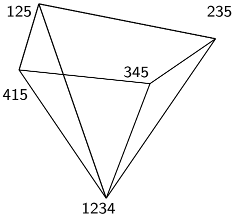
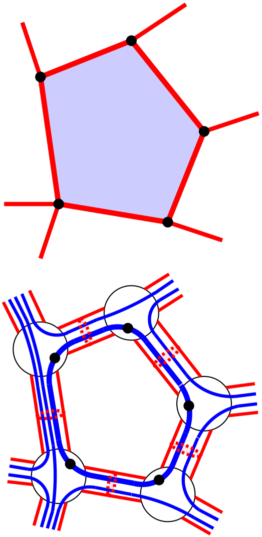




Representations of Polyhedral 2-complexes

LQG/SF: 2-complexes, boundary 1-complexes

- finite abstract 2-polytopes are polygons
- alternative representation: stranded diagrams (faces represented by strands)



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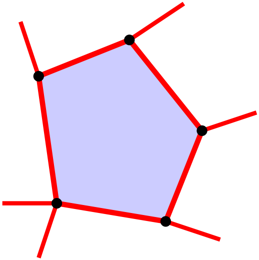

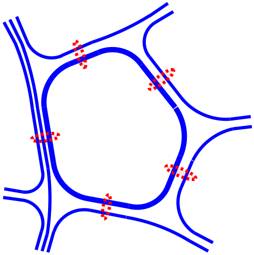

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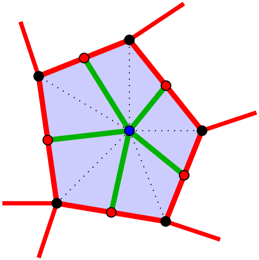

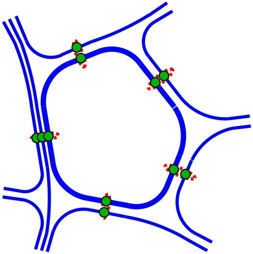

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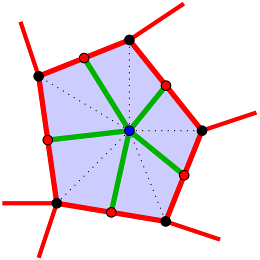
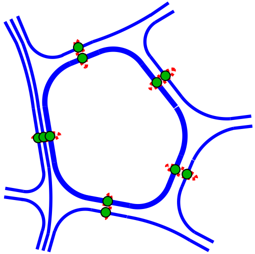
- subdivided cells adjacent to bulk vertex define spin foam atoms





Atomic decomposition

A spin foam atom consists of

- a bulk vertex v
- boundary vertices $\overline{\mathcal{V}} = \{\bar{v}_i, \ldots\}$, isomorphic to bulk edges $(v\bar{v}_i)$
- bisection vertices $\widehat{\mathcal{V}} = \{\hat{v}_{ij}, \ldots\}$, isomorphic to face wedges $(v\bar{v}_i\hat{v}_{ij}\bar{v}_j)$
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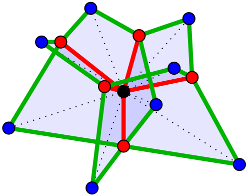

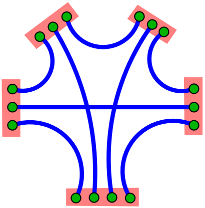

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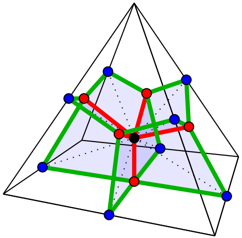
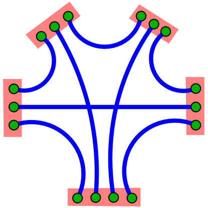
interpretation as dual to (local) D-polytope possible, but not unique





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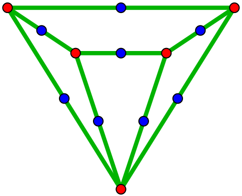
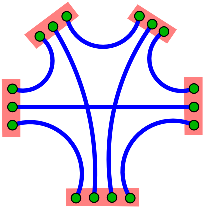
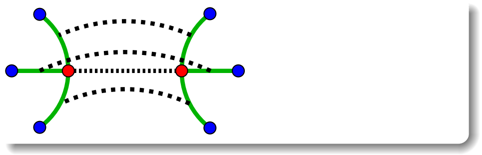




Spin foam atoms are isomorphic to bisected closed graphs (cf. [KKL '10, KLP '12])

(which are isomorphic to closed graphs)

Foams as molecules from atoms



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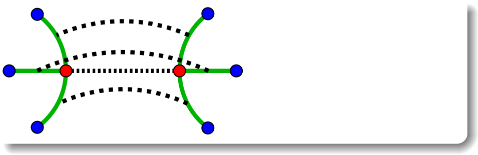

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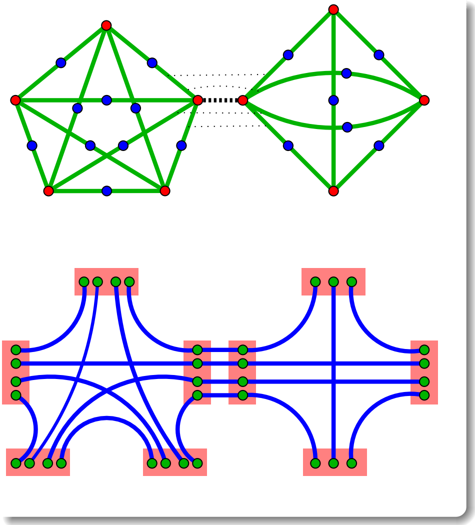

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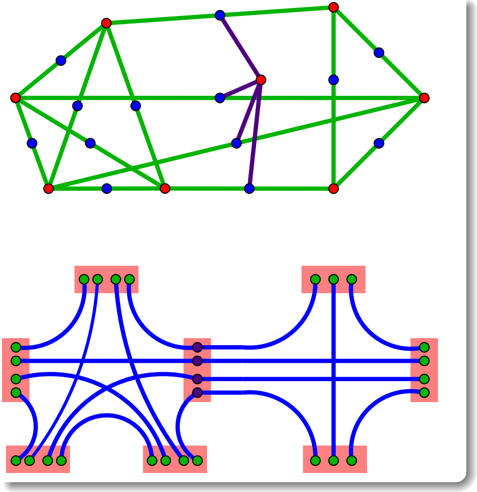

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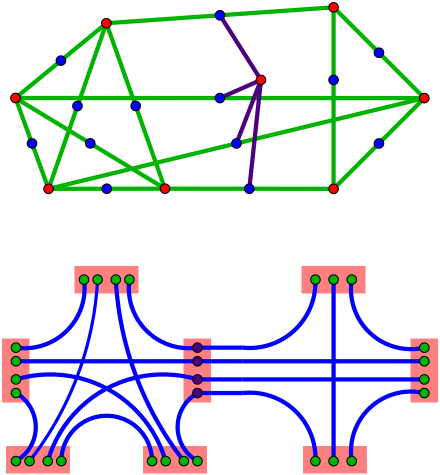

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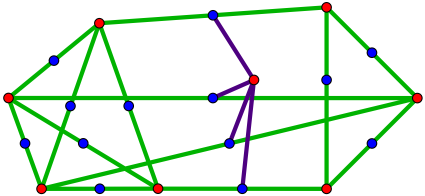

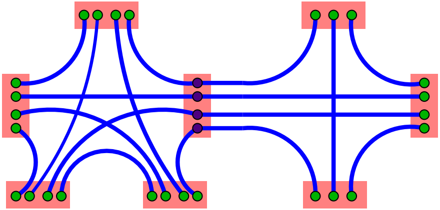

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All of this works for generalized polyhedral complexes of arbitrary dimension

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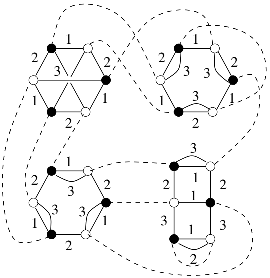

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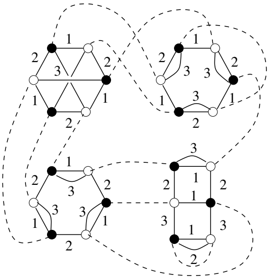
- Decomposition generalizes to GFT without colors: Any regular boundary atoms generated by simplicial interaction



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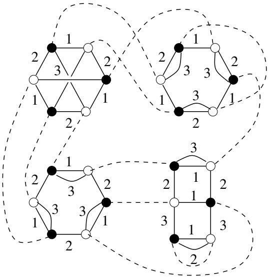
  Ex.: Uncoloring of colored GFT (bipartite simplex gluings) [Bonzom, Gurau, Rivasseau '12]
- Decomposition generalizes to GFT without colors: Any regular boundary atoms generated by simplicial interaction



Prop.: Any atom with regular boundary graph can be decomposed into simplicial atoms

Multi-species GFT

Straightforward generalization to irregular boundary graphs: [Reisenberger, Rovelli '01]

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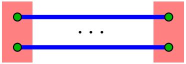

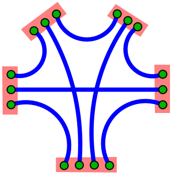

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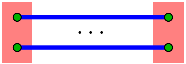

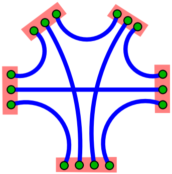

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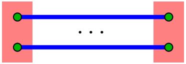

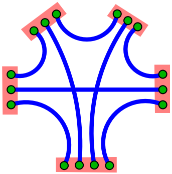

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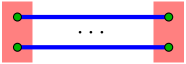

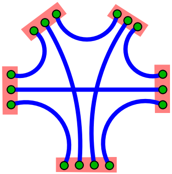

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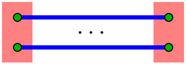

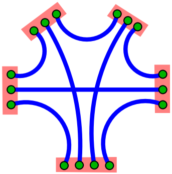

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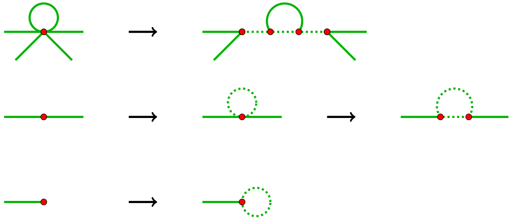
- practical usefullness?

Virtual edges

Correspondence to regular graphs

- k odd: Any graph can be obtained from a k-regular graph by contraction/deletion of edges labeled as virtual
- k even: Any graph with even valencies



Advantage: Only patches with fixed valency needed!

Dually weighted GFT

Fields with indices $m_{\bar{v}\hat{v}} = 0, 1, .., M$: $\phi_{m_{\bar{v}}}(g_{\bar{v}}) = \phi_{m_{\bar{v}\hat{v}_1},...,m_{\bar{v}\hat{v}_D}}(g_{\bar{v}\hat{v}_1}, ..., g_{\bar{v}\hat{v}_D})$

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- using a matrix A, $\lim_{M\to\infty} \frac{1}{M} TrA^q = \delta_{q,2}$: $\mathcal{K}^{(M)}_{m_1 m_2} = \prod_{i=1}^{D} \mathcal{K}^{(M)}_{m_{1,i} m_{2,i}} = \begin{pmatrix} 1 & 0 \\ 0 & A \end{pmatrix}_{m_{1,i} m_{2,i}}$

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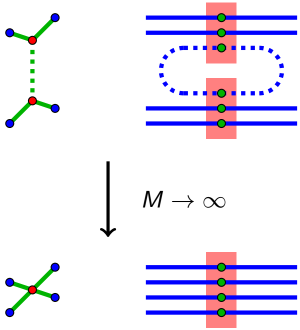

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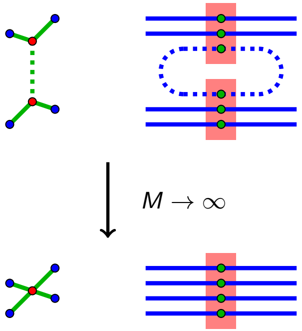

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Result: Virtual edges are not dynamical, propagation as composite objects

Gravitational models, dually weighted

No obstacle for including the established amplitudes

Ex.: EPRL *edge operator* on edge $\bar{v}$:

$$\mathcal{K}(g_{v_1\bar{v}}, g_{v_2\bar{v}}) = \int_{G^{\times 2}} \mathrm{d}h_{v_1\bar{v}} \mathrm{d}h_{v_2\bar{v}} \prod_{(\bar{v}\hat{v})} \sum_{J_{\hat{v}} \in \mathcal{J}} \mathrm{tr}_{J_{\hat{v}}} \left(g_{v_1\bar{v}\hat{v}} h^{-1}_{v_1\bar{v}} \mathcal{S}_{J_{\hat{v}}, N_{\bar{v}}} h_{v_2\bar{v}} g^{-1}_{v_2\bar{v}\hat{v}} \right)$$

($\mathcal{S}_{J,N}$ simplicity operator, $\mathcal{J}$ set of γ–simple reps of $\mathfrak{g} = \mathfrak{so}(4) \cong \mathfrak{su}(2)_+ \times \mathfrak{su}(2)_-$)

- imposes simplicity on "triangulated" atom
- alternative: trivial factor $\delta(g_{v_1\bar{v}\hat{v}} h^{-1}_{v_1\bar{v}}) \delta(h_{v_2\bar{v}} g^{-1}_{v_2\bar{v}\hat{v}})$ on virtual links
- modification of geometricity for higher valency?

Conclusions

- combinatorial compatibility of any LQG and GFT shown
- higher-valent GFT models: multi-species, dual weighting
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Stage is set to analyze these models

- effective interactions, relevant operators
- ...

(though from a QFT perspective the generalization seems rather unnecessary and complicated)