The Strained State Cosmology

Angelo Tartaglia

Politecnico di Torino and INFN

Prologue

- It seems that *something* pushes space to expand, but we do not know what it is. Apparently it does not produce other effects
- It seems that, at a big enough scale, localized gravitational effects exist whose source is not otherwise visible.

Premises

- Apparently the gravitational interaction is very well described as a geometric property of a four-dimensional Riemannian manifold

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

- Other fundamental interactions do not share this geometric essence

Axiomatic assumption

- The physical configuration of the world, in all interactions, satisfies an universal principle of "economy": the 'least' action principle

$$S = \int {}^{*}\Lambda = \int \mathcal{L} d^N x \qquad \delta S = 0$$

Scalar

N-form

Lagrangian density

What is $\mathcal{L}$ made of?

- Formal mathematical answer: *any scalar function of the state variables and their derivatives with respect to the (arbitrarily chosen) coordinates*
- Intuitive physical answer: *in analogy with the Lagrangian densities, a posteriori built starting from the recognized physical laws validated by experiment*

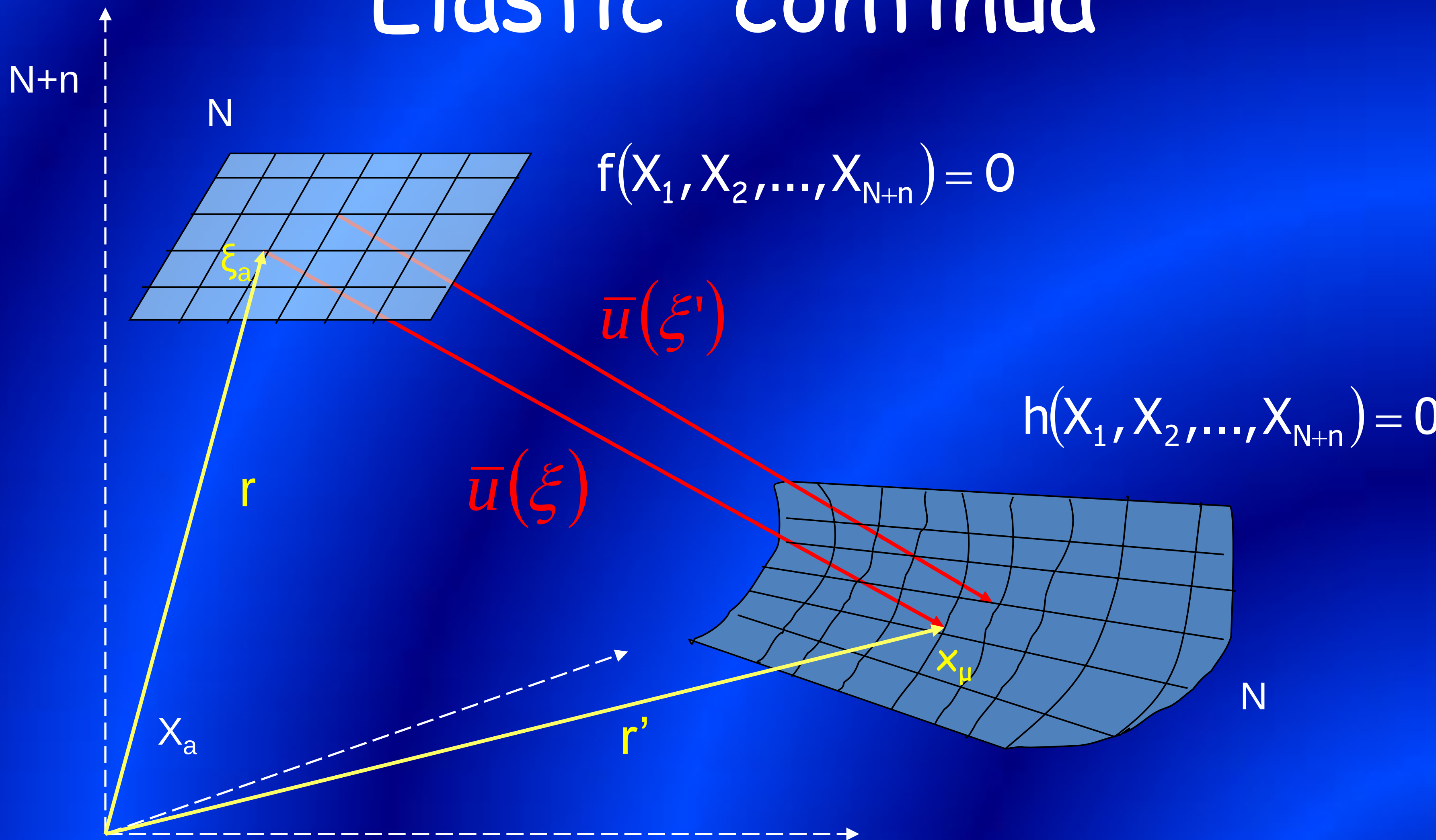
Additional assumption

- The Lagrangian density must have the highest possible 'formal symmetry' (the highest simplicity).
- Non-"simple" functional forms require ad hoc motivations.
  - Is reproducing a specific observed physical situation enough?
  - Is it possible to find universal 'non-simple' solutions?

Tridimensional analogy

- Deformable continuous media
- Geometrizable interaction: elastic interaction (with an external evolution parameter – time -)
- Macroscopic emergent representation, from microscopic elementary interactions

“Elastic” continua



Deformation

- Undifferenciated reference state ≡ flat manifold independent from the parameter.

$$dl_0^{\,2} = E_{ij} dx^i dx^j$$

- Geometry: Euclidean/Minkowskian
- Deformation due either to intrinsic (defects) or 'extrinsic' (matter/energy) causes

$$dl^2 = g_{ij} dx^i dx^j \qquad g_{ij} \text{ globally} \neq E_{ij}$$

- Riemannian geometry

The strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\left(g_{ij} - E_{ij}\right)$$

Lagrangian coordinates

Free energy

$$\mathcal{F} = \mathcal{F}_0 + \frac{\lambda}{2}\left(\varepsilon^{i}{}_{i}\right)^2 + \mu\left(\varepsilon_{ij}\varepsilon^{ji}\right) + ...$$

Lamé coefficients

Second order scalars

The stress (linear elasticity)

$$\sigma^{ij} = C^{ijkl}\varepsilon_{kl} = \lambda E^{ij}\varepsilon + 2\mu\varepsilon^{ij}$$

Hooke's law

$$C_{ijkl} = \lambda E_{ij}E_{kl} + \mu\left(E_{ik}E_{jl} + E_{il}E_{jk}\right)$$

Elastic energy

$$W = \frac{1}{2}\sigma^{ij}\varepsilon_{ij} = \frac{1}{2}\lambda\varepsilon^{2} + \mu\left(\varepsilon_{ij}\varepsilon^{ij}\right)$$

Generalization to 4 dimensions: Lagrangian density

$$S = \int \left[R - \frac{1}{2}\left(\lambda \varepsilon^2 + 2\mu \varepsilon_{\mu\nu}\varepsilon^{\mu\nu}\right) + 2\kappa \mathcal{L}_{matter} \right] \sqrt{-g}\, d^4 x$$

"Kinetic" term

Potential term: "dark energy"

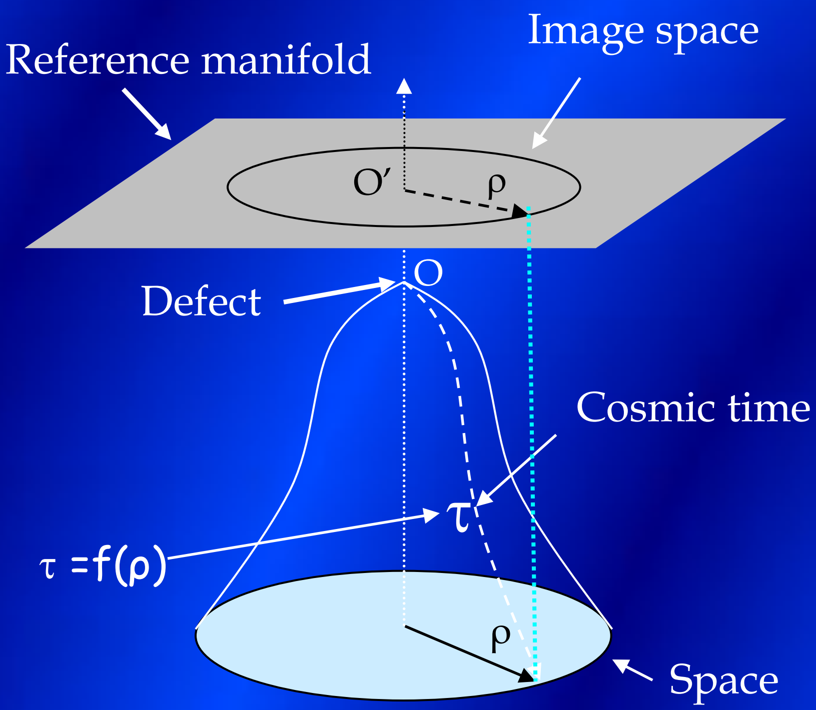
Geometry

Lagrangian density and energy-momentum tensor

$$\mathcal{L} = \left(R - \frac{1}{2}\lambda\left(\varepsilon_{\alpha}{}^{\alpha}\right)^2 - \mu\varepsilon_{\alpha\beta}\varepsilon^{\beta\alpha} \right)\sqrt{|g|}$$

$$T_{(e)\mu\nu} = \lambda\varepsilon\left(\varepsilon_{\mu\nu} - \frac{1}{4}\varepsilon\, g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \right)\sqrt{-g}$$
$$+ \mu\left(\varepsilon_{\mu\alpha}\varepsilon^{\alpha}{}_{\nu} + \varepsilon_{\nu\alpha}\varepsilon^{\alpha}{}_{\mu} - \varepsilon_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\varepsilon_{\alpha\beta}\varepsilon^{\alpha\beta} \right)\sqrt{-g}$$

Robertson-Walker symmetry



$$ds_{ref}^{\ 2} = dr^2 + \alpha dl^2 = b^2(\tau)d\tau^2 + \alpha dl^2$$

$$ds_{nat}^{\ 2} = d\tau^2 - a^2(\tau)\,dl^2$$

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(g_{\mu\nu} - E_{\mu\nu}\right)$$

$$\alpha = \pm 1$$

$$\varepsilon_{00} = \frac{1-b^2}{2} \qquad \varepsilon_{ii} = -\frac{\alpha + a^2}{2}$$

RW Lagrangian density

$$\mathcal{L} = -6\left(a^2\ddot{a} + a\dot{a}^2\right) - \lambda \frac{\left(4a^2 - a^2b^2 + 3\alpha\right)}{8a} - \mu \frac{a^4\left(b^2 - 1\right)^2 + 3\left(a^2 + \alpha\right)^2}{4a}$$

⬇

Integration by parts

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$$\mathcal{L} = 6a\dot{a}^2 - \lambda \frac{\left(4a^2 - a^2b^2 + 3\alpha\right)}{8a} - \mu \frac{a^4\left(b^2 - 1\right)^2 + 3\left(a^2 + \alpha\right)^2}{4a}$$

Euler Lagrange equations

$$\begin{cases} 12a\ddot{a} + 6\dot{a}^2 + \dfrac{3\lambda}{8a^2}\left(a^4\left(b^2-4\right)^2 - 2\alpha a^2 b^2 + 8\alpha a^2 - 3\right) \\ \qquad + \dfrac{3\mu}{4a^2}\left(a^4\left(b^2-1\right)^2 + 3a^4 + 2\alpha a^2 - 1\right) = 0 \\ \dfrac{1}{2}ab\left(\lambda\left(3\alpha + 4a^2 - a^2b^2\right) + 2\mu a^2\left(1-b^2\right)\right) = 0 \end{cases}$$

$$b^2 = 2\frac{B}{\mu} + 3\alpha\frac{\lambda}{\lambda+2\mu}\frac{1}{a^2} \qquad B = \mu\frac{2\lambda+\mu}{\lambda+2\mu}$$

Solution

$$\ddot{a}+\frac{\dot{a}^2}{2a}=-\frac{3}{8}\frac{B}{a^3}\left(a^2+\alpha\right)\left(a^2-\frac{\alpha}{3}\right)$$

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Energy condition

$$6a\dot{a}^2+\frac{3B}{2a}\left(a^2+\alpha\right)^2=W$$

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$$W=0 \quad \rightarrow \quad a=\pm\sqrt{e^{\sqrt{-B}(\tau-T)}-\alpha}$$

The Hubble parameter with matter/radiation

$$H = \frac{\dot{a}}{a} = c\left\{\frac{\kappa}{3}(1+z)^3\left[\rho_{m0} + \rho_{r0}(1+z)\right] - \frac{B}{4}\left(1 + \frac{(1+z)^2}{a_0{}^2}\right)^2\right\}^{1/2}$$

$$\kappa = \frac{8\pi}{c^2}G$$

Equation of state of the "strain fluid"

$$\rho_{(e)} c^2 = -\frac{3}{4} B \frac{\left(a^2 + \alpha\right)^2}{a^4}$$

$$p_{(e)} = \frac{B}{4} \frac{3a^4 + 2\alpha a^2 - 1}{a^4}$$

$$w = -\frac{3a^4 + 2\alpha a^2 - 1}{3\left(a^2 + \alpha\right)^2}$$

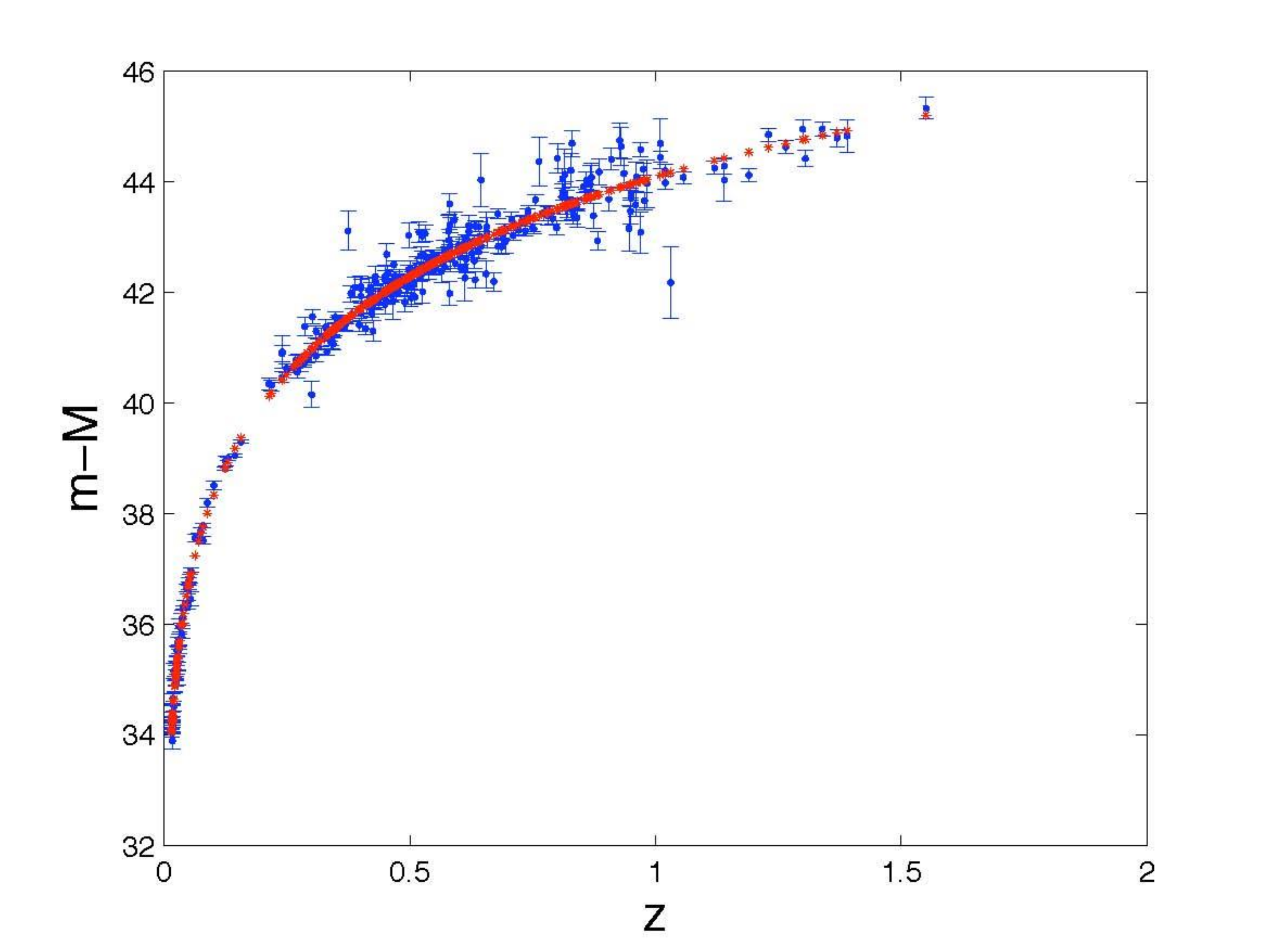
$$w_{a \to 0} \to \frac{1}{3}$$

$$w_{a \to \infty} \to -1$$

Some cosmological tests

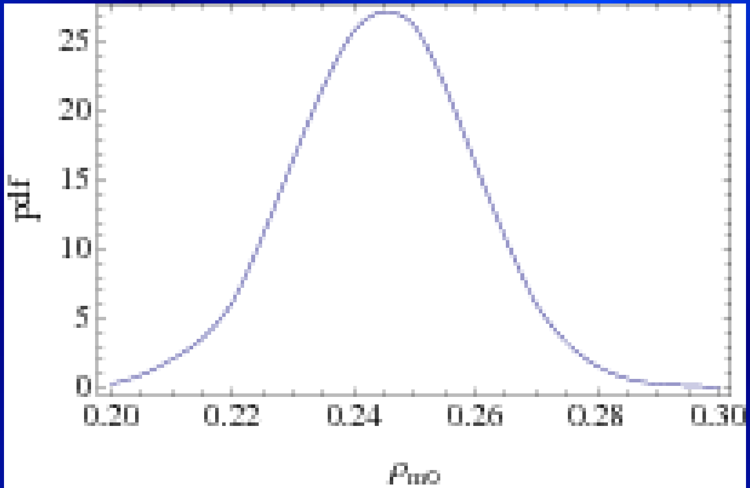
- SnIa luminosity
- Primordial nucleosynthesis (correct proportion between He, D and hydrogen)
- CMB acoustic horizon
- BAO
- Structure formation after the recombination era.

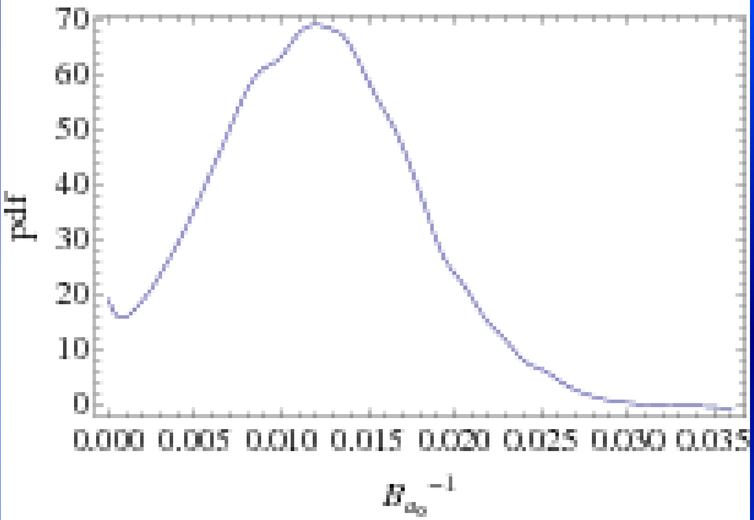
Fitting the supernovae it works

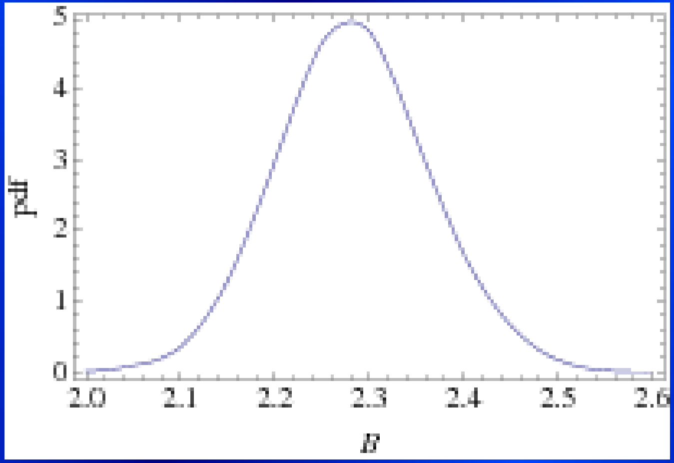


$\lambda \sim \mu \sim 10^{-52} m^{-2}$

Bayesian posterior probability







Optimal value of the parameters

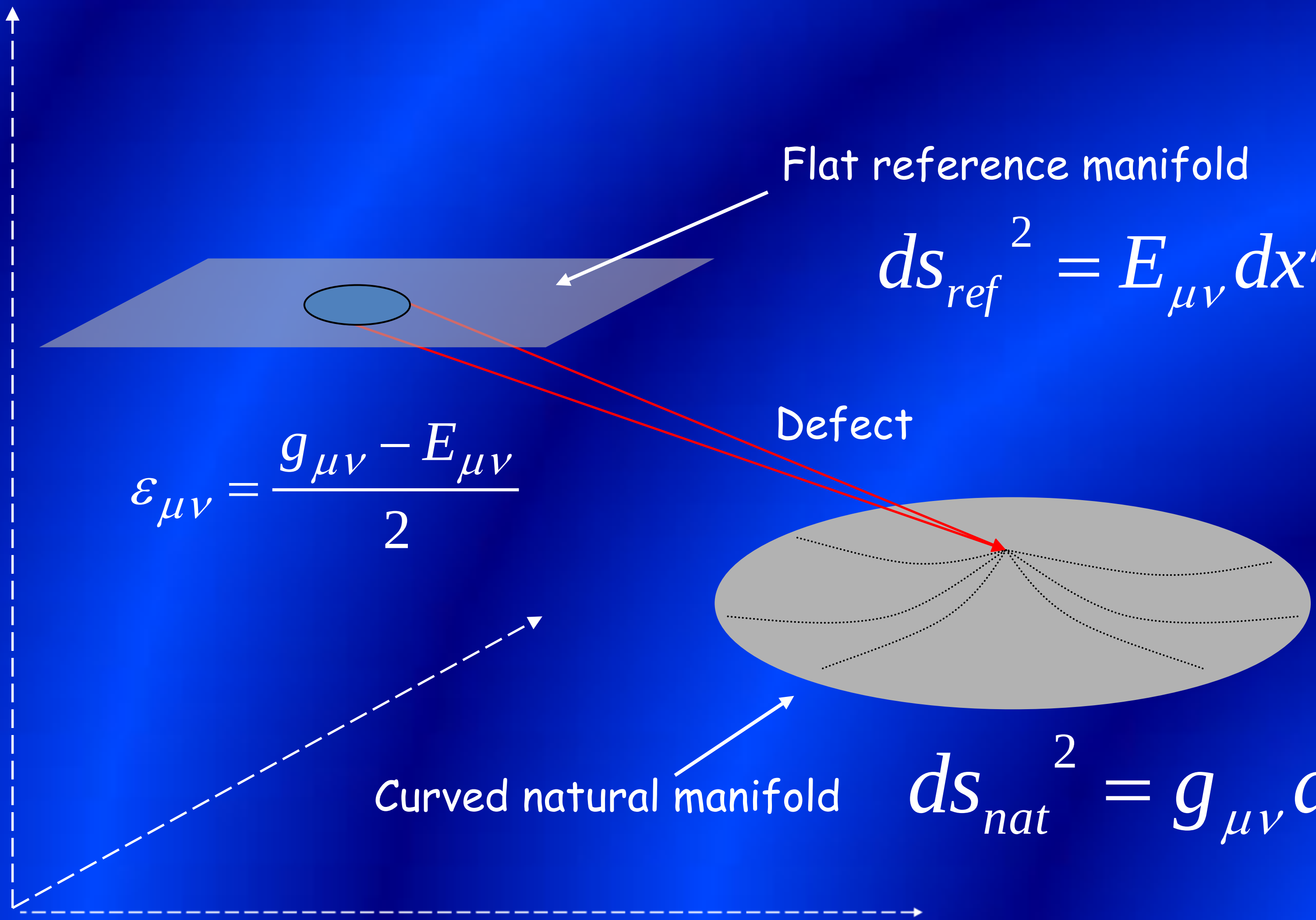
$$B = (2.28 \pm 0.08) \times 10^{-52}\ m^{-2}$$

$$\rho_{m0} = (2.45 \pm 0.15) \times 10^{-27}\ kg/m^3$$

$$B_{a_0}{}^{-1} = (0.012 \pm 0.06) \times 10^{52}\ m^2$$

$$B_{a_0} = \frac{8}{9} \kappa \rho_{r0} a_0{}^4$$

Matter or defects?



$$ds_{ref}{}^{2} = E_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$\varepsilon_{\mu\nu} = \frac{g_{\mu\nu} - E_{\mu\nu}}{2}$$

$$ds_{nat}{}^{2} = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

"Massive" gravity

SST Lagrangian density

$$L_{SST} = \left(\frac{\lambda}{2} \varepsilon^2 + \mu \varepsilon_{\alpha\beta} \varepsilon^{\alpha\beta} \right) \sqrt{-g}$$

Fierz and Pauli Lagrangian density

$$L_{FP} = \frac{m^2}{4} \left(h^2 - h_{\alpha\beta} h^{\alpha\beta} \right)$$

Linearized difference between the metric tensor and a Minkowski background

Non-linear "massive" gravity

Introduce an auxiliary metric tensor $f_{\mu\nu}$ then from it and the background build a quantity $H_{\mu\nu}$ and write

$$L_{CFP} = \frac{m^2}{4}\left(H^2 - H_{\mu\nu}H^{\mu\nu} + \text{higher order terms}\right)\sqrt{-g}$$

"Massive" gravity theories do not correspond to SST where:

- $\varepsilon_{\mu\nu}$ is "exact"
- the only metric tensor is $g_{\mu\nu}$
- the reference manifold is Euclidean

Questions

- Is the elasticity of space-time an emerging property?
  - May be. It involves the issue of the dualism space-time/matter-energy
- Is SST analogous to massive gravity?
  - Not really
- Is SST a bimetric theory?
  - No

Questions

- Can defects get rid of matter?
  - Troubles with quantum mechanics…
- Is this approach better than many others?
  - It depends on which criterion is used to judge what is best.

Conclusion

- The idea is that
  - Space-time is physical
  - there is a deformation energy density in space-time due to curvature.
- If we include a cosmic defect we obtain the Robertson-Walker symmetry and the accelerated expansion.
- SST proposes an intuitive interpretation of Λ, or, more generally, of dark energy.

References

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