Observers diffeomorphism-invariant description of a general relativistic system

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Introduction

- context: canonical General Relativity

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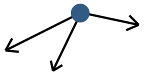
- goal / idea: deparametrise GR with distances and angles



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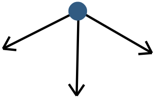

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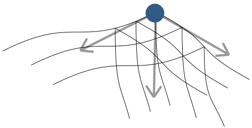

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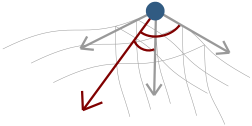

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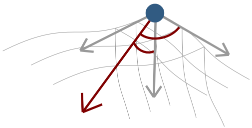

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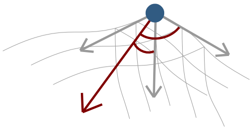

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Note that the above observables are $\mathrm{Diff}_{\mathrm{obs}}$-invariant.

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It turns out that

$$\{F,\ F'\} \rightsquigarrow \text{are canonical!}$$

whereas

$$\{P^{ra},\ F\} \quad \text{and} \quad \{P^{ra},\ P^{rb}\} \quad \text{are nontrivial}$$

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Duch, Kamiński, Lewandowski, J.Ś.
Observables for General Relativity related to geometry JHEP (2014)

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- what do we do with p^{ra}?

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Chapter II: gauge unfixing

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Reduced Phase Space: $q_{AB},\ p^{AB},\ (matter)$

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The hamiltonian constraint turns out to be

$$\int dr\ N \left(\frac{1}{2R^2} \left(\int_0^r dr' \left(P_R R' \right)(r') \right)^2 - \frac{P_R}{R} \int_0^r dr' \left(P_R R' \right)(r') + RR'' + \frac{R'^2}{2} - \frac{1}{2} \right)$$

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Bodendorfer, Lewandowski, J.Ś.
General Relativity in radial gauge – *to appear soon ...*

Thank you for your attention!