Solar wind test of the de Broglie-Proca massive photon with Cluster multi-spacecraft data

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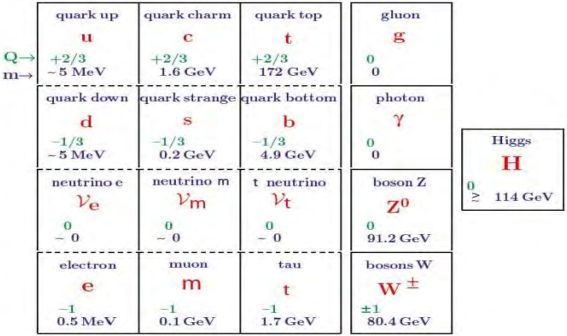
Plan of the talk

- Motivations and considerations.
- The experimental state of affairs.
- The de Broglie-Proca theory.
- Cluster data analysis for the de Broglie-Proca photon (under PRL refereeing).
- Other non-Maxwellian theories. Collaborators: L. Bonetti (Orléans), S. Perez-Bergliaffa and J. Helayël-Neto (Rio de Janeiro).
- Perspectives.

- Our understanding of the universe is largely based on electromagnetic observations (and assumptions).
- As photons are the main messengers, fundamental physics has a concern in testing the foundations of electromagnetism.
- In striking contrast with the complex and multi-parameterised cosmology, electromagnetism is from the 19^{th} century (1826-1867).
- Conversely to the graviton, a mass for the photon isn't frequently assumed.



Investigating non-Maxwellian (nM) theories 2: motivations

Some samples

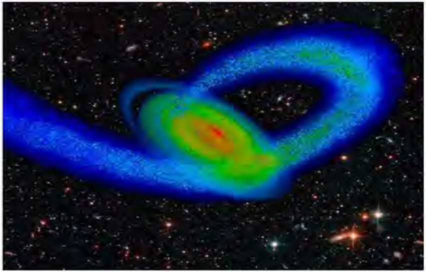
- Hubble constant: 50-100 km/s/Mpc controversy, radioastronomy, Planck data.
- 96% of the universe is unknown.
- And yet, precision cosmology.

nM theories 1: considerations

- Many non-Maxwellian theories following non-linear (Born and Infeld; Heisenberg and Euler) and massive photon theories (de Broglie-Proca).
- Massive photon and yet gauge invariant theories include: Podolsky, Stueckelberg, Chern and Simons.
- Not fashionable but always pursued topic. Four large reviews from 2005.
- Impact on relativity? Difficult answer: variety of the theories above; removal of ordinary landmarks and rising of interwoven implications.
- Experimentalists have mostly conveyed their efforts towards the dBP photon. The upper mass limits of dBP photon mass cannot be generalised to other massive photon theories.
- Impacts on charge conservation and quantisation, magnetic monopoles, superconductors, charged black holes, cosmic microwave background, Higgs' boson, dark matter.

nM theories 2: considerations

The Search for a Massive Photon --"May Reveal Dark Matter and Nix the Standard Model"



Alarcon R., et al., 2013, Phys. Rev. Lett., 111, 164801

"We're looking for a massive photon," explains MIT physics professor Richard Milner. That may seem like a contradiction in terms: Photons, or particles of light, are known to be massless. If it does exist, that would represent a major discovery, Milner says. "It's totally beyond anything we understand about the physical world. A massive photon would be totally different" from anything allowed by the Standard Model, the bedrock of modern particle physics. "It's a tiny effect," Milner adds, but "it can have enormous consequences for our theories and our understanding. It would be absolutely groundbreaking in physics."

TABLE I. A list of the most significant mass limits of various types for the photon and graviton.

Description of method	$\lambda_C \gtrsim$ (m)	$\mu \lesssim$ (eV)	$\mu \lesssim$ (kg)	Comments
1. Secure photon mass limits:				
Dispersion in the ionosphere (Kroll, 1971a)	8×10^{5}	3×10^{-13}	10^{-49}	
Coulomb's law (Williams, Faller, and Hill, 1971)	2×10^{7}	10^{-14}	2×10^{-50}	
Jupiter's magnetic field (Davis, Goldhaber, and Nieto, 1975)	5×10^{8}	4×10^{-16}	7×10^{-52}	
Solar wind magnetic field (Ryutov, 2007)	2×10^{11} (1.3 AU)	10^{-18}	2×10^{-54}	
2. Speculative photon-mass limits:				
Extended Lakes method (Lakes, 1998; Luo, Tu, Hu, and Luan, 2003a, 2003b; Goldhaber and Nieto, 2003)	3×10^{9} $\Leftrightarrow 3\times10^{12}$	7×10^{-7} $\Leftrightarrow 7\times10^{-20}$	10^{-52} $\Leftrightarrow 10^{-55}$	$\lambda_C \sim 4R_\odot$ to 20 AU, depending on **B** speculations
Higgs mass for photon (Adelberger, Dvali, and Gruzinov, 2007)	No limit feasible			Strong constraints on 3D Higgs parameter space
Cosmic magnetic fields (Yamaguchi, 1959; Chibisov, 1976; Adelberger, Dvali, and Gruzinov, 2007)	3×10^{19} (10^{3} pc)	6×10^{-27}	10^{-62}	Needs const **B** in galaxy regions

Experimental limits 2 What about the graviton?

3. Graviton mass limits:

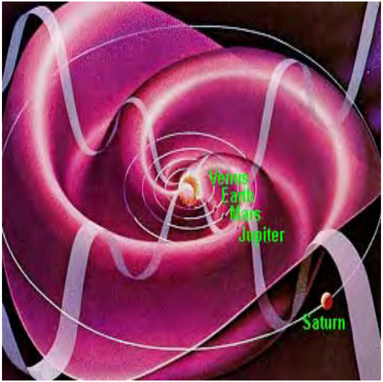
Gravitation wave dispersion (Finn and Sutton, 2002)	3×10^{12}	8×10^{-20}	10^{-55}	Question mark for scalar graviton
Pulsar timing (Baskaran *et al.*, 2008)	2×10^{16}	9×10^{-24}	2×10^{-59}	Fluctuations due to graviton phase velocity
Gravity over cluster sizes (Goldhaber and Nieto, 1974)	2×10^{22}	10^{-29}	2×10^{-65}	
Near field constraints (Gruzinov, 2005)	3×10^{24} (10^{8} pc)	6×10^{-32}	10^{-67}	For DGP model
Far field constraints (Dvali, Gruzinov, and Zaldarriaga, 2003)	3×10^{26} (10^{10} pc)	6×10^{-34}	10^{-69}	For DGP model

Experimental limits 3 dBP photon

- Laboratory experiment (Coulomb's law) 2×10^{-50} kg.
- Dispersion-based limit $3 \cdot 10^{-49}$ kg (lower energy photons travel at lower speed). Note: some quantum gravity theories foresee the opposite (Amelino-Camelia).
- Ryutov finds $m_\gamma < 10^{-52}$ kg in the solar wind at 1 AU, and $m_\gamma < 1.5 \times 10^{-54}$ kg at 40 AU (PDG value). These values come partly from *ad hoc* models. Limits: (i) the magnetic field is assumed exactly always and everywhere a Parker's spiral; (ii) the accuracy of particle data measurements (from e.g. Pioneer or Voyager) has not been discussed; (iii) there is no error analysis..
- More speculative, lower limits from modelling the galactic magnetic field: 10^{-62} kg.
- Modelling of hydromagnetic waves in Crab Nebula give ten orders of magnitude difference between analysis carried by different research groups (Barnes, Scraggle, Phys. Rev. Lett., 1975; Chibisov, Sov. Phys. Usp., 1976).
- Newer limits from black holes stability (Pani et al., Phys. Rev. Lett., 2012); CPT violation (Dolgov, Novikov, Phys. Lett. B, 2014) are theoretical limits.

Experimental limits 4: Parker's spiral

As the Sun rotates, its magnetic field twists into an Archimedean spiral, as it extends through the solar system. This phenomenon is named after Eugene Parker's work: he predicted the solar wind and many of its associated phenomena in the 1950s. The spiral nature of the heliospheric magnetic field had been noted earlier by Hannes Alfvén.



Experimental limits 5 Goldhaber and Nieto, Rev. Mod. Phys., 2000

Quote

"Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterisations. This is perhaps due to the temptation to assert too strongly something one knows to be true. A look at the summary of the Particle Data Group (Amsler et al.. 2008) hints at this. In such a spirit, we give here our understanding of both secure and speculative mass limits."

Experimental limits 6

The lowest theoretical limit on the measurement of any mass is dictated by the Heisenberg's principle $m \geq \hbar \Delta t c^2$, and gives 3.8×10^{-69} kg, where Δt is the supposed age of the Universe. The same principle implies that measurements of masses in the order of 10^{-54} kg should be performed in time scales of at least thirty minutes.

de Broglie-Proca (dBP) theory 1

- The concept of a massive photon has been vigorously pursued by Louis de Broglie from 1922 throughout his life. He defines the value of the mass to be lower than 10^{-53} kg. A comprehensive work of 1940 contains the modified Maxwells equations and the related Lagrangian.
- Instead, the original aim of Alexandru Proca, de Broglie's student, was the description of electrons and positrons. Despite Proca's several assertions on the photons being massless, his Lagrangian (1936) and formalism (1937) apply to a massive real or complex vector field.
- Theories and conjectures centered on massive photons have been later proposed by several authors.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \underline{\frac{1}{2}m^2 A_\mu A^\mu} + j^\mu A_\mu \tag{1}$$

dBP equations (SI units) where $F_{\mu\nu} = \partial_m u A^\nu - \partial_\nu A^\mu$.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2 \phi \ , \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ , \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \ , \tag{4}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A} \ , \tag{5}$$

ϵ_0 permittivity, μ_0 permeability, ρ charge density, $\vec{j}$ current, ϕ and $\vec{A}$ potential.
$\mathcal{M} = 2\pi m_\gamma c/h = 2\pi/\lambda$, h Planck constant, c speed of light, λ Compton's wavelength, m_γ photon mass.
Eqs. (2, 5) are Lorentz-Poincaré transformation but not Lorenz gauge invariant.
In a static regime (Lorenz = Coulomb gauges), Eqs. (2, 5) are not coupled through the potential. $\nabla \cdot \vec{A} + \partial\phi/\partial t = 0$.

- dBP wave equation implies slower speeds for lower frequencies

$$\left[\partial_\mu \partial^\mu + \underline{\left(\frac{m_\gamma c}{\hbar}\right)^2}\right] A^\nu = 0 \qquad (6)$$

- For $m_\gamma \neq 0$, the speed of propagation depends upon the frequency. At sufficiently high frequencies, for which the photon rest energy is small with respect to the total energy, the difference in velocity for two different wavelengths λ is

$$\Delta v = v_{g1} - v_{g2} = \frac{m_\gamma^2 c^3}{8\pi^2 \hbar^2}(\lambda_2^2 - \lambda_1^2) \qquad (7)$$

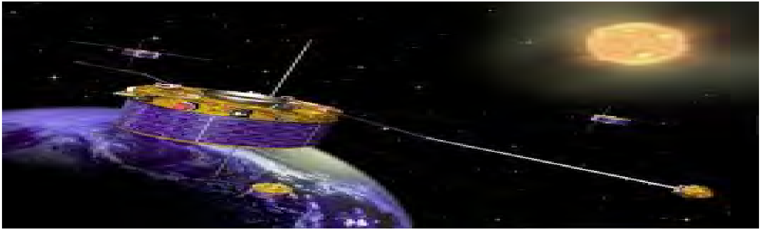
being v_g the group velocity.

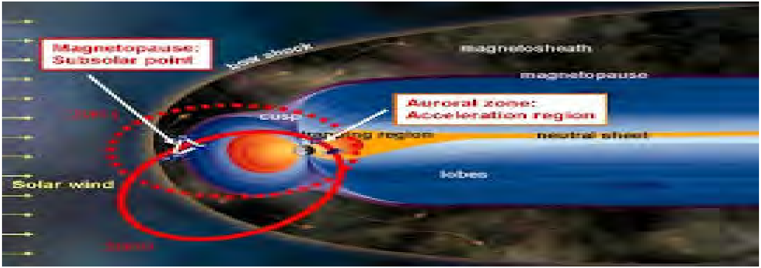
- For a single source at distance d, the difference in the time of arrival of the two photons is

$$\Delta t = \frac{d}{v_{g1}} - \frac{d}{v_{g2}} \simeq \frac{\Delta v d}{c^2} = \frac{d m^2 c}{8\pi^2 \hbar^2}(\lambda_2^2 - \lambda_1^2) \qquad (8)$$

- Such behaviour reproduces interstellar dispersion the delay in pulse arrival times across a finite bandwidth. Dispersion occurs due to the frequency dependence of the group velocity of the pulsed radiation through the ionised components of the interstellar medium. Pulses emitted at lower radio frequencies travel slower through the interstellar medium, arriving later than those emitted at higher frequencies.
- In absence of an alternative way to measure plasma dispersion, there is no way to disentangle plasma effects from a dBP photon.
- Assuming arrival times only due to plasma dispersion, the most stringent limit comes from the results of several pulsar measurements throughout the visible, near infrared and ultraviolet regions of the spectrum 3×10^{-49} kg (Bay, White, Phs. Rev. D, 1972), whereas from a single pulsars the limit is 8.4×10^{-49} kg (Bhat et al., Ap. J., 2004).

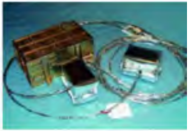
Cluster data analysis 1: the mission





Cluster data analysis 2: the instruments





Cluster data analysis 3: the philosophy

- Such small mass induces to extreme caution: precise experiment or very large apparatus.
- The largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind. For this purpose, we evaluate the dBP modified Ampère's law.
- Cluster (ESA): 4 spacecraft flying in tetrahedral configuration at 1 AU from the Sun, and having variable inter-spacecraft separation ranging from 10^2 to 10^4 km.
- Cluster has allowed for the first time the direct computation of three-dimensional quantities such as $\nabla \times \vec{B}$ from magnetic field measurements; this was not possible with earlier spacecraft.
- Cluster carries also particle detectors.

Since we are interested in the large-scale steady components of the magnetic field, *i.e.* to very low frequencies, the displacement current density in Eq. (5) can be dropped: indeed

$$\epsilon_0\mu_0\frac{\partial E}{\partial t}\sim\epsilon_0\mu_0\frac{Ev_{sw}}{L_B}\sim\epsilon_0\mu_0\frac{Bv_{sw}^2}{L_B}\sim\underline{2\times 10^{-22}\ \mathrm{Am}^{-2}}\ ,$$

being $v_{sw} = 4 \times 10^2$ km s^{-1} the typical solar wind velocity, and L_B the caracteristic length of the magnetic field.

The dbP modified Ampère's law reads

$$\boxed{\nabla \times \vec{B} = \mu_0\vec{j} - \mathcal{M}^2\vec{A}}\, . \qquad (9)$$

Cluster data analysis 5: the philosophy

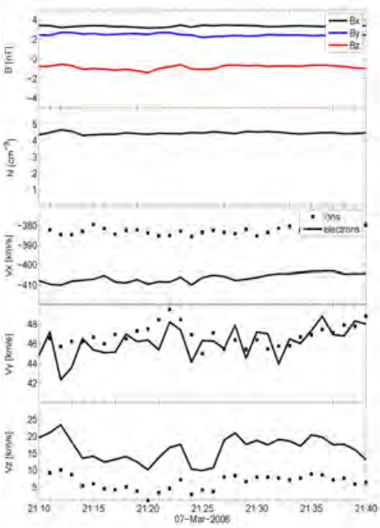
- For $\vec{j}_B = \nabla \times \vec{B}/\mu_0$ and $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$, n the number density, e the electron charge, $\vec{v}_i$, $\vec{v}_e$ the velocity of the ions and electrons, respectively, the dBP photon mass is

$$\boxed{m_\gamma = \frac{k}{\left|\vec{A}_H\right|^{\frac{1}{2}}}\left|ne(\vec{v}_i - \vec{v}_e) - \frac{\nabla \times \vec{B}}{\mu_0}\right|^{\frac{1}{2}} = \frac{k\left|\vec{j}_P - \vec{j}_B\right|^{\frac{1}{2}}}{\left|\vec{A}_H\right|^{\frac{1}{2}}}\,,} \tag{10}$$

  where $k = \hbar\mu_0^{\frac{1}{2}}c^{-1}$, and $\vec{A}_H$ is the vector potential from the interplanetary magnetic field.
- Event selection to compare with PDG (1 AU) limit: (i) an *undisturbed* solar wind, *i.e.* disconnected from the terrestrial bow shock, far from the terrestrial H; (ii) the closest location of the spacecraft to the equatorial plane; (iii) the widest inter-spacecraft separation, 10^4 km, assuring the largest differences in H among the spacecraft; (iv) the configuration best approaching the tetrahedron; (v) the availability of good quality particle currents.

Cluster data analysis 6: the current from curl B

- $B_x > 0$, $B_y > 0$ and $B_x, B_y \gg B_z$, as expected for a Parker's spiral configuration close to the ecliptic plane.
- However, our analysis does not rely on the Parker's model, since the magnetic field is measured *in situ*. The conditions are similar to those presented by Ryutov (1997, 2007), official PDG limit, for comparison.
- We measure j_B by using the curlometer from the Cluster fluxgate magnetometer. This method allows to compute the average $\nabla \times \vec{B}$ over the tetrahedron with no assumptions on the field analytical form (only assuming linear gradients) and to assess the error on j_B.
- Similar results on the error on $< j_B >$, through a second and independent procedure, were acquired. The method is based on applying random independent variations on magnetic field measurement and satellite position at each of the satellites, and estimating the standard deviation of the obtained current fluctuations.



Panel (a). The three components of the magnetic field for Cluster 3 in the GSE (Geocentric Solar Ecliptic) coordinate system. Panel (b). The average plasma density. Panels (c,d,e).

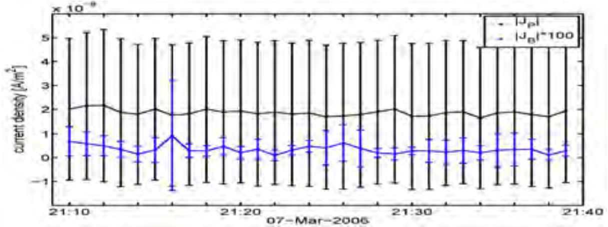
The v_x, v_x, v_x velocity components of ions (dotted line) and electrons (full line).

Cluster data analysis: vector potential 8

- The average of the vector potential due to the interplanetary magnetic field $< A_H >$ at Cluster location is computed from j_B.
- The characteristic length of the magnetic field is $L_B \sim < B > / \mu_0 j_B = < B > / |\nabla \times \vec{B}/| = 9.6 \times 10^4$ km, where $< B >$ is the average magnetic field over the tetrahedron. For this event, the inter-spacecraft separation is $L \approx 6 \times 10^3$ km.
- $< A_H > \sim < B > \times L_B \approx 4.1 \times 10^{-1}$ T m.

Cluster data analysis 8: particle current

- The particle current density $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$ from ion and electron currents; n is the number density, e the electron charge and $\vec{v}_i$, $\vec{v}_e$ the velocity of the ions and electrons, respectively.
- An accurate assessment of the particle current density in the solar wind is difficult due to inherent instrument limitations.
- $j_P >> j_B$ (up to four orders of magnitude), mostly due to the differences in the i, e velocities, while the estimate of density is reasonable. While we can't exclude that this difference is due to the dBP massive photon, the large uncertainties related to particle measurements hint to instrumental limits.



- For the event considered, $j_P = 1.86 \cdot 10^{-7} \pm 3 \cdot 10^{-8}$ A m^{-2}, while $j_B = |\nabla \times \vec{B}|/\mu_0$ is $3.5 \pm 4.7 \cdot 10^{-11}$ A m^{-2}.

$$\boxed{\begin{aligned} m_\gamma &< \frac{k}{A_H{}^{\frac{1}{2}}}\left[(j_P - j_B)^{\frac{1}{2}} + \frac{\Delta j_P + \Delta j_B}{2(j_P - j_B)^{\frac{1}{2}}} + \frac{\Delta A_H (j_P - j_B)^{\frac{1}{2}}}{2A_H}\right] \\ &\simeq \frac{k}{A_H{}^{\frac{1}{2}}}\left[j_P{}^{\frac{1}{2}} + \frac{\Delta j_P}{2(j_P)^{\frac{1}{2}}}\right] \simeq \frac{k}{A_H{}^{\frac{1}{2}}}[j_P + \Delta j_P]^{\frac{1}{2}} \approx 1.4 \times 10^{-49}\ \mathrm{kg}\ . \end{aligned}} \tag{11}$$

WARNING: the analysis will be reopened for A_H (refereeing), but a very low estimate on A_H impacts not more than 40%, that is 2.1×10^{-49} kg. A_H is an estimate, not a measurement.

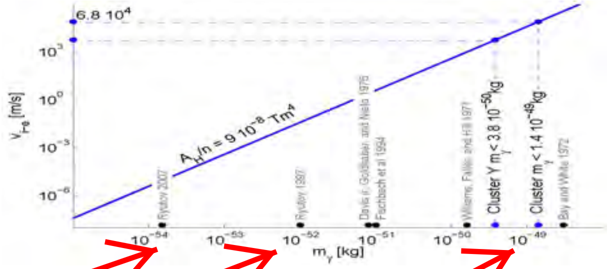
Largest A_H (estimate!) leads to less than 2 orders of magnitude lower m_gamma

Most stringent limitation comes from particle detectors. The difference between ion and electron velocities is

$$v_{i-e} \sim \frac{(j_P + \Delta j_P)}{ne} \approx \underline{6.8 \times 10^4 \ \mathrm{m\ s^{-1}}} \tag{12}$$

$n = 4.46 \times 10^6$ m^{-3} ion density.
By recasting Eq. (11) as $v_{i-e}(m_\gamma)$, we derive the minimum v_{i-e} that particle detectors should measure to resolve a given upper bound for m_γ.

- The marks refer to laboratory, dispersion, planetary and solar wind limits of earlier literature and to our Cluster spacecraft test. In our study $A_H/n \approx 9 \times 10^{-8}$ T m^4.
- The upper limit 10^{-52} kg reported by Ryutov 1997 in the solar wind at 1 AU would require resolving a difference $v_{i-e} \approx 10^{-1}$ ms^{-1}, that is not possible with currently available particle detectors onboard Cluster and other spacecraft. This is almost six orders of magnitude difference with respect to the Cluster event studied here.



Cluster data analysis 12: possible improvements

- Consider only the z component.
- Set artificially but justifiably $j_p = j_B$. How? a) Confidence on previous literature results; b) difference between ion and electron velocities cannot be very large.
- We would improve of a factor 100.
- Ultimately, a technological revolution for particle detectors.

Cluster data analysis 13: results

- A zero cost experiment based a non-dedicated mission leads to a result just one order of magnitude lower than ground experiment.
- We have reported a new approach to estimate the dBP photon mass. We have found larger values than previous solar wind estimates, our test being based on fewer assumptions.
- We do not assume that the interplanetary magnetic field is a Parker's spiral, though we have chosen events compatible to the Parker's spiral for comparison (Ryutov, 1997, 2007).
- Only solar wind test considering in detail the experimental errors.
- Confirmation of the de Broglie's prediction (1922) on m_γ upper limit.
- The domain between our findings ($m_\gamma < 1.4 \times 10^{-49}$ kg) and the results from *ad-hoc* model in the solar wind ($m_\gamma < 1.5 \times 10^{-54}$ kg) is still subjected to assumptions and conjectures, though fewer now, and not to clear-cutting outcomes from experiments. Our experiment is limited by the resolution of the velocity difference between ions and electrons.

Other non-Maxwellian (nM) theories 1: considerations

- A review of the thirty ? nM theories is not available in the literature, neither theoretical nor experimental review.
- Most experimental tests related to set upper limits to dBP photon mass.
- Strangely, Maxwellian-like equations are often not displayed.
- On-going work: L. Bonetti (Orléans), S. Perez-Bergliaffa and J. Helayël-Neto (Rio de Janeiro). We show the forefathers' theories (Stueckelberg, Podoiski, Born-Infeld, Euler-Heisenberg).

Other non-Maxwellian (nM) theories 2: Stueckelberg

- The Stueckelberg Lagrangian

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2\left(A_\mu - \frac{\partial_\mu B}{m}\right)^2 - (\partial^\mu A_\mu + mB)^2 \tag{13}$$

  where B is a scalar field to render the dBP *manifestly* gauge invariant.

- We have two fields and two equations of motion. The wave equations are

$$\partial_\mu\partial^\mu A^\nu + m^2 A^\nu = 0 \tag{14}$$

$$\partial_\mu\partial^\mu B + m^2 B = 0 \tag{15}$$

- First massive photon theory, gauge invariant

$$A_\mu \rightarrow A_\mu + \partial_\mu\Lambda \qquad B \rightarrow B + m\Lambda \qquad (\partial^2 + m^2)\Lambda = 0$$

- Used as alternative to dark energy, Akarsu et al., 2014 arXiv:1404.0892.

Other non-Maxwellian (nM) theories 3: Podolsky

- The Podolsky Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{b^2}{4}\left(\partial^\nu F^{\mu\nu}\right)\partial_\nu F_{\mu\nu} + j^\mu A_\mu \qquad (16)$$

  where b has the dimension of m^{-1}.

- The equations are

$$-b^2\partial_\mu\partial^\mu\left(\vec{\nabla}\cdot\vec{E}\right) + \vec{\nabla}\cdot\vec{E} - \rho = 0 \qquad (17)$$

$$-b^2\partial_\mu\partial^\mu\left[\frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B}\right] + \frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B} + \vec{j} = 0 \qquad (18)$$

- Gauge invariant $A_\mu \rightarrow A_\mu + \partial_\mu\Lambda$
- Magnetic monopoles? and massive photons.
- Cut-off for short distances $\phi = \frac{e}{4e\pi}(1 - e^{-\frac{r}{b}})$

Other non-Maxwellian (nM) theories 4: Born-Infeld

- The Born-Infeld Lagrangian

$$\mathcal{L} = \sqrt{1+F} - 1 + j^{\mu} A_{\mu} \tag{19}$$

- The equations are

$$\partial_{\mu}\left(\frac{F^{\mu\nu}\left(1+F\right)^{-\frac{1}{2}}}{2}\right) = j^{\nu} \tag{20}$$

- Electromagnetic mass. The mass is derived from the field energy.
- Avoidance of infinities out of self-energy $\phi = \frac{e}{r_0} f\left(\frac{r}{r_0}\right)$
- The parameter b poses a limit to the electric field (to be understood).

Other non-Maxwellian (nM) theories 5: Euler-Heisenberg

- The Euler-Heisenberg Lagrangian

$$\mathcal{L} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{e^2}{\hbar c}\int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3}\cdot\left\{ i\frac{\eta^2}{2}F^{\mu\nu}F^*_{\mu\nu}\cdot \right.$$
$$\cdot\frac{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F^*_{\mu\nu}}\right] + \cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F^*_{\mu\nu}}\right]}{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F^*_{\mu\nu}}\right] - \cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F^*_{\mu\nu}}\right]}$$
$$\left. + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6}\cdot F_{\mu\nu}F^{\mu\nu}\right\} \qquad (21)$$

$$F^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \qquad \mathfrak{E}_k = \frac{m^2c^3}{e\hbar} \sim 10^{16}\frac{V}{m} \qquad (22)$$

$\mathfrak{E}_k$ critical field for creating electron-positron pairs from vacuum.

- Light-Light scattering.
- Particle creation on cosmological scale (Starobinsky and others).
- Photon splitting.
  http://www.nature.com/news/2010/100728/full/news.2010.381.html

Perspectives

- Analysis of other nM theories with Cluster.
- Application of non-linear theories to magnetars.
- Radiation reaction in Born-Infeld theory (singularity free).
- Light propagation in nM theories.

Grazie