A Test Bed For High Energy Physics[1]

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Abstract

We briefly comment upon the parallel between graphene and high energy fermions and explore the possibility of using the former as a test bed for the latter rather like Reynold's numbers in a wind tunnel. We also point out that there are parallels to Quantum Gravity approaches, which indeed provide a novel explanation for such

effects as the FQAE.

0.1 Introduction

In two dimensions and one dimension electrons will display strange neutrino like properties as had been pointed out by the author, starting the mid nineties [1, 2, 3]. The

two component equation that is obeyed [4] is

$$\left(\sigma^\mu \partial_\mu - \frac{mc}{\hbar}\right) \psi = 0 \tag{1}$$

where σ^μ denote the 2×2 Pauli matrices. In case the mass vanishes (1) gives the neutrino equation. This has relevance to graphene that was discovered nearly a decade later. For the electron quasi particles in graphene we have

$$\nu_F \vec{\sigma} \cdot \vec{\nabla} \psi(r) = E \psi(r) \tag{2}$$

$\nu_F \sim 10^6 m/s$ **is the Fermi velocity replacing** c**, the velocity of light and** $\psi(r)$ **being a two component wave function** E **denoting the energy.**

In any case Landau had shown several decades ago that such two and one dimensional structures would be unstable and as such cannot exist – and this was proved wrong.

In any case anomalous behaviour would be expected for Fermions in low dimensions.

This is because spin $\frac{1}{2}$ **(unlike for bosons) is in some sense an entanglement with the**

ambient three dimensions (Cf.ref.[5] for a detailed description). Once this support is not available, we can expect, in low dimensions such anomalous, neutrino like behaviour.

However, there is no Lorentz invariance (except in the case of a hypothetical infinite sheet) and the two component wave function $\psi(r)$ in (2) comes from the wave functions in two side by side honey comb lattices. We will see this later. This is rather like spin

up and spin down.

0.2 Graphene As A Test Bed

We now point out that graphene can be a test bed for high energy physics. Firstly (2) represents a neutrino like (massless) Fermion. Indeed the massless feature has been experimentally confirmed. These are quasi particles. If we consider bi-layer graphene

then even the mass comes in.

Secondly graphene behaves like a "chess board", in the sense that there are space gaps between the carbon atoms and lattices. That is there is a minimum "length" [6]. So a non-commutative geometry holds.

In this case we have

$$[x_\imath, x_j] = \Theta^{\imath j} l^2 \tag{3}$$

where as can be seen the coordinates $x_\imath$ and x_j do not commute l is the distance between lattices. As a result of this the Maxwell equations get modified with an extra term, as shown in detail elsewhere [7, 8]:

$$\partial^\mu F_{\mu\nu} = \frac{4\pi}{c} j_\nu + A_\lambda \epsilon F_{\mu\nu} \tag{4}$$

where the symbols have their usual meaning. In (4) ε is a dimensionless number which is equal to one for our non-commutative case namely (3), and is zero otherwise.

With $\varepsilon = 0$ we get back the usual covariant Maxwell equations. Specializing to two dimensions we get

$$\partial^1 F_{14} = \frac{4\pi}{c} j_4 + A_2 \varepsilon F_{14} \tag{5}$$

and similar equations for the j_1 and j_2. In this case, using the electromagnetic tensor we get equations like

$$\frac{\partial E_x}{\partial x} = -4\pi \frac{\partial \rho}{\partial t} + \varepsilon A_y E_x \tag{6}$$

$$-\frac{\partial B_z}{\partial x} = 4\pi j_y + \epsilon \frac{\partial E_y}{\partial t} \tag{7}$$

and similar equations. These show that we are dealing with non-steady fields which give radiation.

This clearly brings out the extra electromagnetic effects. Because of (3) there appears a magnetic field as was shown by the author and Saito [9, 10]. We can deduce the

equation

$$Bl^2 = hc/e \tag{8}$$

In the case of Graphene, keeping in mind the somewhat different values for the constants like ν_F and l, we would have

$$Bl^2 = h\nu_F/e.$$

The energy in the above is linear and given by

$$\textbf{Energy} = \pm\nu_F|\vec{p}|$$

The positive sign denotes conduction and the negative sign valence particles, the analogues of particles and antiparticles.

The analogy with high energy physics, particularly in the Cini-Toushek regime is very strong (Cf.ref.[11]). There too, we encounter a massless scenario. In fact at very high

energies we have [11]

$$H\psi = \frac{\vec{\alpha} \cdot \vec{p}}{|p|} E(p) \tag{9}$$

which resembles the massless equation (1). In (9) we have

$$\alpha^k = \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{10}$$

$$\gamma^0 = \beta \tag{11}$$

This can be readily generalized to the neutrino equation.

However there are differences with the usual Dirac Theory – here we do not encounter Lorentz invariance and ν_F is not the velocity of light, rather its analogue some three hundred times less.

We can see that Graphene will be a test bed in some interesting situationsin the sense of Reynolds numbers in wind tunnels. The author had already argued several years

ago [12, 13] that for nearly monoenergetic Fermions or even Bosons there would be a loss of dimensionality and the collection would behave as if it were in two dimensions. This immediately mimics the two dimensional feature.

Our starting point is the well known formula for the occupation number of a Fermion gas[14]

$$\bar{n}_p = \frac{1}{z^{-1} e^{bE_p} + 1} \tag{12}$$

where, $z' \equiv \frac{\lambda^3}{v} \equiv \mu z \approx z$ **because, here, as can be easily shown** $\mu \approx 1$,

$$v = \frac{V}{N}, \lambda = \sqrt{\frac{2\pi\hbar^2}{m/b}}$$

$$b \equiv \left(\frac{1}{kT}\right), \quad \textbf{and} \quad \sum \bar{n}_p = N \tag{13}$$

Let us consider in particular a collection of Fermions which is somehow made nearly mono-energetic, that is, given by the distribution,

$$n'_p = \delta(p - p_0)\bar{n}_p \tag{14}$$

where $\bar{n}_p$ is given by (12).

This is not possible in general - here we consider a special situation of a collection of mono-energetic particles in equilibrium which is the idealization of a hypothetical experimental set up.

By the usual formulation we have,

$$N = \frac{V}{\hbar^3}\int d\vec{p}n'_p = \frac{V}{\hbar^3}\int \delta(p-p_0)4\pi p^2\bar{n}_p dp = \frac{4\pi V}{\hbar^3}p_0^2\frac{1}{z^{-1}e^{\theta}+1} \tag{15}$$

where $\theta \equiv bE_{p_0}$.

It must be noted that in (15) there is a loss of dimension in momentum space, due to the δ function in (14).

Similarly, recently the author had pointed out that the neutrinos behaved as if they were a two dimensional collection. Indeed [15] one could expect this from the holographic principle. Equally the author (and A.D. Popova) had argued that the universe

itself is asymptotically two dimensional [16].

Furthermore it has also been argued that not only does the universe mimic a Black Hole, but also that the Black Hole is a two dimensional object [17, 18]. Indeed the interior of a Black Hole is in any case inaccessible and the two dimensionality follows from the area of the Black Hole which plays a central role in Black Hole Thermodynamics. The author had shown, in his analysis that the area of the Black Hole is given

by

$$A = Nl_p^2 \tag{16}$$

For these Quantum Gravity considerations we have to deal with the Quantum of area [19, 18]. In other words we have to consider the Black Hole to be made up of N Quanta of area. Thus we can get an opportunity to test these Quantum Gravity features in two dimensional surfaces such as graphene.

In the earlier communication [20] it was shown that in the one dimensional case, corresponding to nanotubes we would have

$$kT = \frac{3}{5} kT_F \tag{17}$$

where T_F is the Fermi temperature. It can be seen that for the two dimensional case too kT is very small. This is because using the well known formulae for two dimensions

we have

$$kT = \frac{e\hbar\pi}{m\nu_F} \tag{18}$$

$$(kT)^3 = \frac{6e\hbar\nu_F}{\pi} \tag{19}$$

Whence we have

$$(kT)^2 = 6 \cdot \nu_F^2 \pi^2 m \tag{20}$$

Remembering that $\nu_F \sim 10^8$, we have even for a particle whose mass is that of an electron, from (20) kT is very small. For a comparison we have for the Fermi temperature,

$$kT_F = \frac{\hbar}{2}(z6\pi)^{1/3} \cdot \nu_F$$

Another conclusion which could have been anticipated is the following. We have from the above

$$\nu_F^2 = \left(\frac{\hbar\pi}{m}\right)^2 \cdot \frac{1}{A} \tag{21}$$

where $A \sim l^2$ is the quantum of area. So we get

$$\frac{m^2\nu_F^2}{\hbar^2} \cdot l^2 \sim 0(1) \tag{22}$$

This is perfectly consistent with ν_F tending to the velocity of light c and $h/m\nu_F$ tending to the Compton wavelength. In other words an infinite graphene sheet would give us back the usual spacetime of Relativity and Quantum Mechanics. In practise we could expect this for a very large sheet of graphene. In either case it turns out that whatever

be the temperature, it is as if the ensemble behaves like a very low temperature gas.

This leads to many possibilities, particularly about magnetism.

As pointed out above we can investigate magnetism and electromagnetism in this new non-commutative paradigm which throws up novel features including the Haas Van Alphen type effect [7]. In this case, the magnetization per unit volume, as is known, shows an oscillatory type behaviour.

0.3 Discussion

Fluctuations of the Zero Point Field have been widely studied. Based on this the author in 1997 predicted a contra model of the universe [21, 18] in which there would be a small cosmological constant, that is an accelerating universe. In 1998 observations of Perlmutter, Reiss and Schmidt confirmed this scenario. Today we call this dark energy. A manifestation of this is a noncommutative spacetime given in (3). This lead to the

so called Snyder-Sidharth dispersion relation with an extra energy term (Cf.ref.[18]).

We would like to point out that the extra magnetic effect in equations like (6) (and the following) can be attributed to this Zero Point effect of noncommutativity as given in (8). Closely related is the Casimir effect which has been observed even in graphene [22, 23]. This is a Zero Point Field fluctuation effect. The Casimir energy in graphene

is given by

$$\frac{Energy}{area} = \frac{\pi^2}{240} \cdot \frac{\hbar c}{a^3} \tag{23}$$

The energy itself is given by

$$Energy = \left(\frac{\pi^2}{240}\right) \cdot \frac{\hbar c}{a} \tag{24}$$

where we consider the area to be $\sim a^2$.

If following Wheeler [5] we consider directly ground state oscillators of the Zero Point

Field, we can deduce that

$$Energy \sim \hbar c/a$$

resembling (24). Similarly if we take the extra energy term in the dispersion relation as described in ref.[18], it is easy to show that this also has the same form. All this is hardly surprising because they are all manifestations of fluctuations in the Quantum Vacuum.

It must be mentioned that the Casimir effect in graphene has been observed. What is interesting is that a group of scientists from MIT, Harvard University, Oak Ridge National Laboratory and other Universities have used this Zero Point energy for a compact integrated silicon chip. Clearly the same would be possible for graphene too particularly in the context of Quantum Computers: the "Spin" up and down being the qubits [24].

To proceed further we invoke (8) and the well known result for a coil

$$\imath = \frac{NBA}{R\Delta t} \tag{25}$$

where N **is the number of turns,** A **is the area and** R **the resistance. Use of (8) in (25)**

now gives

$$\imath \approx \frac{NA}{R} \cdot \frac{e}{l^2 \tau} \tag{26}$$

Whatever be N, if we think of a coil made up of nanotubes or graphene, remembering that l is small and so is the resistance (26) would be observable, like indeed (8). Further observing that nanotubes and graphene can harbour fast moving Fermions (including neutrons) and of course carbon, we have all the ingredients for manipulating a version of table top fusion possibly using the bosonization of fermions property. In this case we could use an equation like (15) and preceding consideration [18, 20].

To proceed, in this case, $kT =< E_p >\approx E_p$ so that, $\theta \approx 1$. But we can continue without giving θ any specific value.

Using the expressions for v and z given in (13) in (14), we get

$$(z^{-1}e^{\theta} + 1) = (4\pi)^{5/2}\frac{z'^{-1}}{p_0}; \textbf{whence}$$

$$z'^{-1}A \equiv z'^{-1}\left(\frac{(4\pi)^{5/2}}{p_0} - e^{\theta}\right) = 1, \tag{27}$$

where we use the fact that in (13), $\mu \approx 1$ as can be easily deduced.

A number of conclusions can be drawn from (27). For example, if,

$$A \approx 1, i.e.,$$

$$p_0 \approx \frac{(4\pi)^{5/2}}{1+e} \tag{28}$$

where A is given in (27), then $z' \approx 1$. Remembering that in (13), λ is of the order of the de Broglie wave length and v is the average volume occupied per particle, this means

that the gas gets very densely packed for momenta given by (28). Infact for a Bose gas, as is well known, this is the condition for Bose-Einstein condensation at the level $p = 0$ (cf.ref.[14]).

In any case there is an anomalous behaviour of the Fermions.

0.4 Final Comments

One of the mysteries involving graphene is that there is a minimum conductivity which does not disappear. This conductivity is given by

$$\sigma = 4e^2/\hbar \tag{29}$$

However this mystery is easily solved if we remember that as seen earlier, because of (3), that is noncommutativity of space in the hexagonal two dimensional crystal. We

have (8) or as seen from equations (6) and (7) there is an extra electric and magnetic field. To proceed, we have for the electron mobility μ and conductivity σ:

$$\mu = \nu_F / |E| \tag{30}$$

$$\sigma = (n/A)e \cdot \frac{\nu_F}{|E|},\ A \sim l^2 \tag{31}$$

The minimum of $n \sim 1$. Whence we get $(|B| = |A|)$

$$\sigma = \frac{1}{l^2} \cdot \frac{e \cdot \nu}{|B|} \tag{32}$$

Here l, the analogue of the Compton length is the inter lattice distance.

From (32) we can easily obtain (29) on using (8) or the subsequent equation. In other words the mysterious minimum conductivity is due to the extra magnetic effect of noncommutative spacetime which holds for graphene.

If we consider in (31) n to be the number of electrons in general and A to be the area of

m **honeycomb lattices, then we can get from (31) the fractional Quantum Hall Effect:**

$$\sigma = \frac{n}{m} \cdot \frac{e^2}{h}$$

It is surprising that effects like FQHE are the manifestation of non-commutative space considerations which mimic Quantum Gravity results.

0.5 Appendix

In a recent paper [25] we had argued that two dimensional crystals like graphene can be a possible test bed for High Energy Physics experiments as behavior like zitterbewegung, Compton scale , non commutative space time etc. are exhibited though at a different scale much like wind tunnels work with Reynolds scaled down numbers.

For example, the Fermi velocity replaces the velocity of light, which is some 300 times

higher.

It was also pointed out that such effects as minimum conductivity and the Fractional Quantum Hall Effect get a remarkable derivation due to the non-commutative nature of the space of these structures.

We would like to point out two new and important points in this context. The first is that the magnetic field in this case is stronger than the usual Maxwellian field [26]

and in fact is now given by its expression in non commutative space:

$$Bl^2 = \frac{hc}{e}. \tag{33}$$

In equation (33), symbols have their usual meaning except that l stands for the minimum length, the lattice length in this case. This was deduced independently by the author and Saito several years ago [27, 28, 29]. The experimentally observed and

mysterious minimum conductivity is given by

$$\sigma = 4\frac{e^2}{h} \tag{34}$$

Again symbols have their usual meanings in equation (34). Remarkably (34) can be deduced from the above considerations [25]. What is equally remarkable is that the magnetic field (33) and the electric current following from (34) arise solely as a result of the non commutative space geometry of these two dimensional crystalline structures.

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