Quantum Gravity and the Foundations of Quantum Theory

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Motivation

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In the foundations of modern **classical physics**, **time and space** are dynamical entities on an equal footing.

Not so in **quantum theory**. A **predetermined notion of time** enters in an essential way in the standard description of the measurement process. The **noncommutativity** at the very heart of quantum theory arises in the comparison of measurements with different **temporal** order. This makes quantum theory seemingly **inapplicable in a context that lacks a background time**, such as general relativity.

Into the foundations

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Or is this an artifact of non-relativistic thinking in the founding days of quantum mechanics? Is there an underlying timeless formulation of quantum theory?

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Or is this an artifact of non-relativistic thinking in the founding days of quantum mechanics? Is there an underlying timeless formulation of quantum theory?

Two approaches to investigate the second possibility:

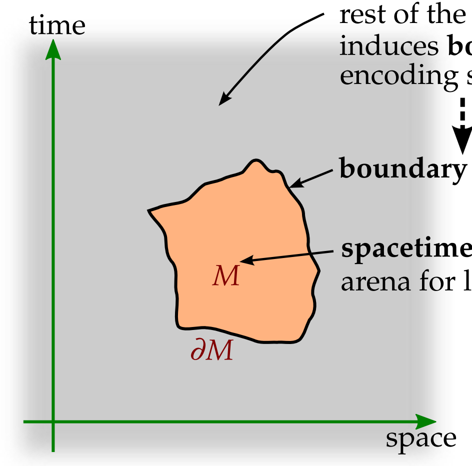
- **Examine known quantum physics** (in particular quantum field theory) with a view towards uncovering deeper underlying structure.
- **Forget** about the concrete form of quantum theory as we know it. Instead, reason about the **general structure an operational description** of nature could or should have.

Usually: First approach. **Today: Second approach.**
Surprise: Convergence!

Guidelines

- **Locality**: We have learned that to understand and describe local physics, a knowledge or control of the immediate spatial and temporal surroundings is sufficient.
- **Operationalism**: In classical physics sweeping statements about physical reality independent of a possible observer are possible and even sensible. This is not so in quantum theory. Rather, we should be describing physics through the interaction with an observer or experimenter.

Locality and spacetime



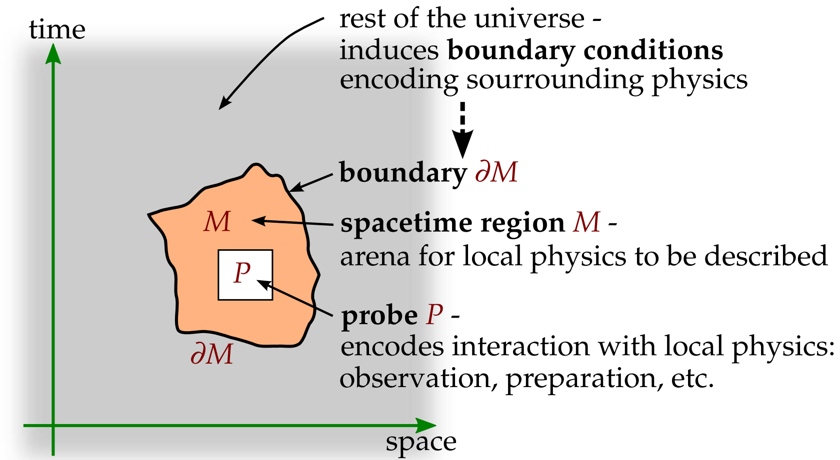
rest of the universe - induces **boundary conditions** encoding sourrounding physics

boundary ∂M

spacetime region M - arena for local physics to be described

Require a **notion of spacetime**:
spacetime regions and their **boundaries**.

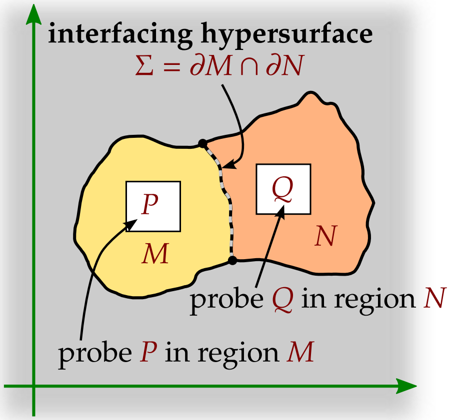
Probes



A **probe** is associated to a spacetime region.
There is also a special **null-probe** representing the absence of a probe.

Composition

For a comprehensive description it is essential that we be able to relate the physics in adjacent spacetime regions.



Need an operation that allows to **combine probes** P, Q in adjacent spacetime regions M, N to a composite probe $P \diamond Q$ in the joint region $M \cup N$.

"Holography"

Information about local physics is communicated between adjacent regions through **boundary conditions** on **interfacing hypersurfaces**.

Towards a quantitative description of physics

Associate mathematical structures to the ingredients identified so far.

- For a **region** M we denote the space of **probes** in M by $\mathcal{P}_M$. We denote the null-probe by $\nu \in \mathcal{P}_M$. The composition of probes is a map $\diamond : \mathcal{P}_M \times \mathcal{P}_N \to \mathcal{P}_{M \cup N}$.
- To a **hypersurface** Σ we associate a space $\mathcal{B}_\Sigma$ of **boundary conditions**. This encodes the possible physical information flows between the two regions adjacent to the hypersurface Σ.
- To a **probe** P in a **spacetime region** M with **boundary condition** $b \in \mathcal{B}_{\partial M}$ we associate a **quantity**. We shall take this to be a **real number** and denote it by $(P, b)_M$. It encodes a property of the local physics in the interior as detected by the probe and subject to the boundary condition. Formally, $(\cdot, \cdot)_M : \mathcal{P}_M \times \mathcal{B}_{\partial M} \to \mathbb{R}$.

Quantities

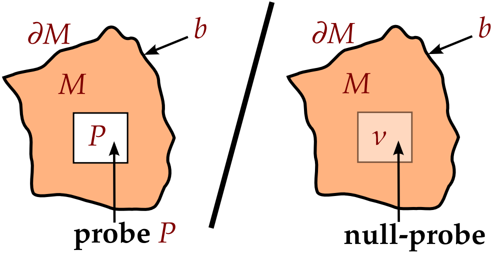
The quantities $(P, b)_M$ might be

- **direct** or **relative**: Direct quantities have an immediate physical interpretation. Relative ones need to be combined (usually two).
- **deterministic** or **probabilistic**: Deterministic quantities correspond to measurement outcomes. Probabilistic ones only yield probabilities or expectation values.

Direct quantities from relative quantities

If quantities are only relative, we need to **relate** different probes and/or boundary conditions in order to extract physical quantities. The quantities obtained are then **conditional** relations.

Simplest case: Condition on a **boundary condition** b by comparing to the **null-probe**.

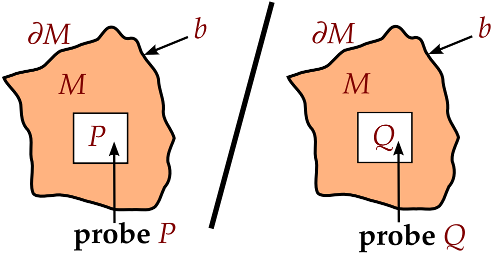


$$\frac{(P, b)_M}{(\nu, b)_M}$$

is the measurement outcome of probe P in M given boundary condition b

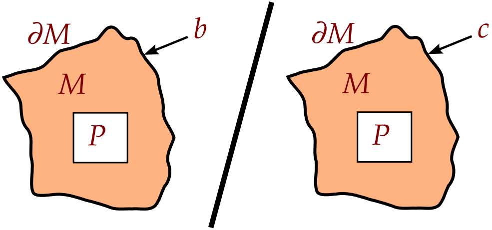
Direct quantities from relative quantities

Probes and or **boundary conditions** may form **hierarchies** encoded in **partial orderings** that facilitate the extraction of conditional relations.



$$\frac{(P,b)_M}{(Q,b)_M}$$

outcome of measurement $P \leq Q$ given apparatus Q in M with boundary conditions b.



$$\frac{(P,b)_M}{(P,c)_M}$$

outcome of measurement P for boundary condition $b \leq c$ given that the class of boundary conditions c is present.

Encoding classical physics

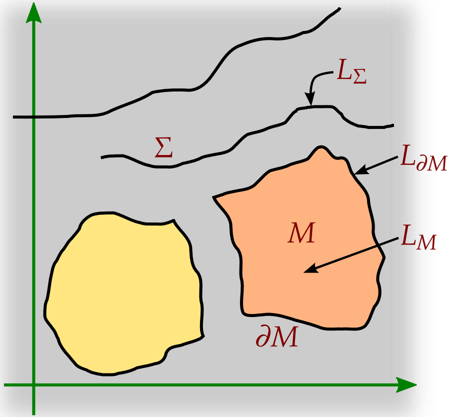
Before considering a generic setting we show that

1. **classical (deterministic) physics**
2. **classical statistical physics**

can be fit into this framework. We consider these in turn.

Classical physics

In classical field theory we are naturally given the following structures:

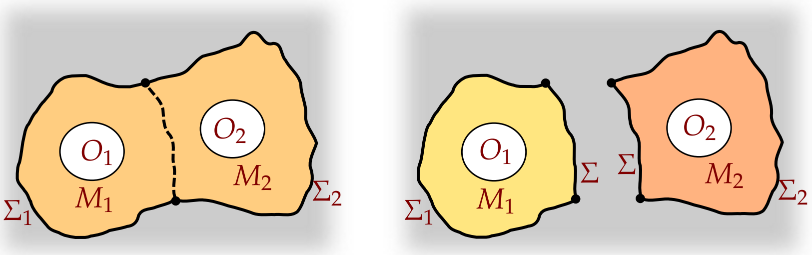


- Per **hypersurface** Σ :
  The **space** L_Σ **of solutions** near Σ.
- Per **region** M :
  The **space of solutions** in M. Forgetting the interior yields a map $L_M \to L_{\partial M}$.

Observables

In classical physics the role of **probes** is taken by **observables**. An observable in a region M is a function $O : L_M \to \mathbb{R}$.

Consider regions M_1, M_2 with matching boundary components Σ and their **composition** to a joint region $M = M_1 \cup M_2$.



The joint observable $O = O_1 \diamond O_2$ is the product

$$O(\phi) = O(\phi|_{M_1}) \cdot O(\phi|_{M_2})$$

where $\phi \in L_M$.

Physical quantities

- A **boundary condition** on Σ is a boundary solution, i.e., $\mathcal{B}_\Sigma = L_\Sigma$.
- For a spacetime region M and boundary condition $\varphi \in L_{\partial M}$ the quantity for the **null-probe** is,

$$(\nu, \varphi)_M := \begin{cases} 1 & \text{if there is } \phi \in L_M \text{ with } \varphi = \phi|_{\partial M} \\ 0 & \text{otherwise} \end{cases}$$

  This is the truth-value of whether a given boundary condition can be physically realized or not.
- For a spacetime region M a **probe** is an **observable** O in M. To the boundary condition φ assign the quantity,

$$(O, \varphi)_M := \begin{cases} O(\phi) & \text{if there is } \phi \in L_M \text{ with } \varphi = \phi|_{\partial M} \\ 0 & \text{otherwise} \end{cases}$$

  If the boundary condition is physically realizable this yields the value of the observable.

Statistical classical physics

- We consider **boundary conditions** that are **probability densities** μ on the space $L_{\partial M}$ of boundary solutions (which may be thought of as **statistical ensembles**).
- As before, **probes** are **observables**. Given an observable O in the spacetime region M with boundary condition μ we define the associated quantity as,

$$(\nu, \mu)_M := \int_{L_M} O(\phi)\, \mu(\phi|_{\partial M})$$

Examples of physical quantities:

$$(\nu, \mu)_M$$

is the fraction of the boundary probability distribution μ that is physically realizable.

$$\frac{(O, \mu)_M}{(\nu, \mu)_M}$$

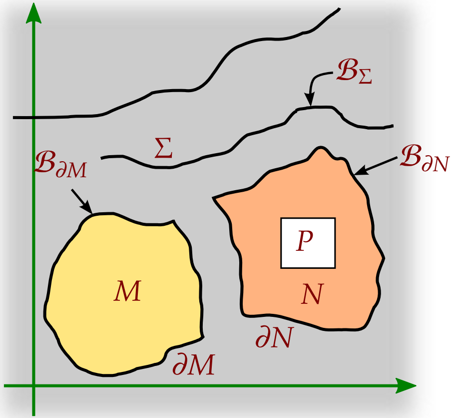
is the expectation value of O given the probability distribution induced by the boundary condition μ.

A probabilistic setting

Consider a setting where quantities are relative and give rise to **probabilities** and **real expectation values**. (Just like in the classical statistical setting.)

- The spaces $\mathcal{B}_\Sigma$ of boundary conditions are **real vector spaces with a partial order**.
- A class $\mathcal{P}^+_M$ of **basic probes** (including the null-probe) on M give rise to **values** that are **positive linear functions** on $\mathcal{B}_{\partial M}$. (This is required for relative probabilities.)
- All **probes** on M give rise to **values** that are **real linear functions** on $\mathcal{B}_{\partial M}$. The space $\mathcal{P}_M$ of probes on M itself is a **real vector space with a partial order**.

Spacetime assignments



To the geometric structures associate the data,

- per **hypersurface** Σ : an **ordered vector space** $\mathcal{B}_\Sigma$,
- per **region** M : a **positive linear map** $(v, \cdot)_M : \mathcal{B}_{\partial M} \to \mathbb{R}$,
- per **region** M that contains a **probe** P : a **real linear map** $(P, \cdot)_M : \mathcal{B}_{\partial M} \to \mathbb{R}$.

Physical quantities

- Given boundary conditions $b \leq c \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{(v, b)_M}{(v, c)_M}$$

  is the **conditional probability** for b to be realized given c.

- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition c is given by,

$$\frac{(P, c)_M}{(v, c)_M}.$$

Physical quantities – of quantum theory!

- **Boundary conditions** generalize **mixed states** and **projection operators**.
- **Probes** generalize **observables** and **weighted quantum operations**.
- Given boundary conditions $b \le c \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{(v,b)_M}{(v,c)_M}$$

  is the **conditional probability** for b to be realized given c. **Transition amplitudes** arise as a special cases of this.
- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition c is given by,

$$\frac{(P,c)_M}{(v,c)_M}.$$

  Conventional **expectation values** arise as special cases of this.

Quantum theory

It turns out that a formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

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It turns out that a formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

This is called the **general boundary formulation of quantum theory**.

The key point is that the extraction and coherent interpretation of physical quantities in this formulation does not require any notion of time. (But it does require a weak notion of spacetime.)

This suggests a suitable basis for implementing quantum theories in a generally covariant setting.

General boundary formulation

So far, there exist two versions of this:

- The **amplitude formalism**: generalizes Hilbert spaces, amplitudes, observables
  - based on the mathematical framework of **topological quantum field theory** (TQFT) [Witten, Segal, Atiyah, . . . 1988–]
  - can be equipped with present physical interpretation [RO 2005]
- The **positive formalism**: generalizes spaces of mixed states, super operators, quantum operations
  - arises as "modulus square" of amplitude formalism, leads to "positive TQFT" [RO 2012]

The formulation we have arrived at can be identified precisely with the **positive formalism**.