Primordial non-Gaussianity: present status and future prospects

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Going beyond the Gaussian hypothesis in Cosmology

Historical remarks

• Groth and Peebles 1977 (3-pt function)
• Strongly non-Gaussian initial conditions studied in the eighties
• Determination of bispectrum for PSCz galaxies (Fedman et al. 2001) 2dF galaxies (Verde et al. 2002)
• New era with $f_{NL}$ non-Gaussian (NG) models from inflation (Salopek & Bond 1991; Gangui et al. 1994: $f_{NL} \sim 10^{-2}$; Verde et al. 1999; Komatsu & Spergel 2001; Acquaviva et al. 2002; Maldacena 2002; + many models with higher $f_{NL}$).
• Primordial NG gradually emerged as a new “smoking gun” of (non-standard) inflation models, which complements the search for primordial gravitational waves.
Non-Gaussianity & Initial Conditions
NG probes the physics of the Early Universe

- The NG amplitude and shape measures deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...
- Inflation models which would yield the same predictions for scalar spectral index and tensor-to-scalar ratio might be distinguishable in terms of NG features.
- We should aim at “reconstructing” the inflationary action, starting from measurements of a few observables (like $n_S$, $r$, $n_T$, $f_{NL}$, $g_{NL}$, etc. ...), just like in the nineties we were aiming at a reconstruction of the inflationary potential (see revival of this industry after Bicep2 ...).
Simple-minded NG model

Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the simple formula (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 1999; Komatsu & Spergel 2001)

$$\Phi = \phi_L + f_{NL} \left( \phi_L^2 - <\phi_L^2> \right) + g_{NL} \left( \phi_L^3 - <\phi_L^2> \phi_L \right) + ...$$

where $\Phi$ is the large-scale gravitational potential (more precisely $\Phi = 3/5 \, \zeta$ on superhorizon scales, where $\zeta$ is the gauge-invariant comoving curvature perturbation), $\phi_L$ its linear Gaussian contribution and $f_{NL}$ the dimensionless non-linearity parameter (or more generally non-linearity function). The percent of non-Gaussianity in CMB data implied by this model is

$$\text{NG} \% \sim 10^{-5} \, |f_{NL}|$$

$$\sim 10^{-10} \, |g_{NL}|$$

“non-Gaussian = non-dog”
(Ya.B. Zel’dovich)

< $10^{-4}$ from CMB & LSS

< $10^{-4}$ from CMB & LSS
NG shape information

NG is measured by the bispectrum (FT of 3-point function), whose amplitude is regulated by $f_{NL}$

... there are more shapes of non-Gaussianity (from inflation) than ... stars in the sky

bispectrum shapes

1. Squeezed
2. Equilateral
3. Folded

(1) Squeezed (2) Equilateral (3) Folded

local shape: Multi-field models, Curvaton, Ekpyrotic/cyclic, etc. ....
equilatral shape: Non-canonical kinetic term, DBI, K-inflation, Higher-derivative terms, Ghost, EFT approach
orthogonal shape: Distinguishes between variants of non-canonical kinetic term, higher-derivative interactions, Galilean inflation, EFT
flat shape: non-Bunch-Davies initial state and higher-derivative interactions, models where a Galilean symmetry is imposed, EFT The flat shape can be written in terms of equilateral and orthogonal.
Non-Gaussianity &
Cosmic Microwave Background (CMB)
The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.
Planck 2013 results XXIV: Planck collaboration: arXiv:1303.5084

Scientific target

• Constrain (with high precision) and/or detect primordial non-Gaussianity (NG) as due to (non-standard) inflation (NG amplitude and shape measure deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...)

• We tested: local, equilateral, orthogonal shapes (+ many more) for the bispectrum and constrained the primordial trispectrum (test of multi-field models) parameter $\tau_{NL}$
CMB bispectrum

\[ B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \equiv \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \]

\[ = \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} b_{\ell_1 \ell_2 \ell_3} \]

Gaunt integrals

\[ \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3} \equiv \int Y_{\ell_1 m_1}(\hat{n}) Y_{\ell_2 m_2}(\hat{n}) Y_{\ell_3 m_3}(\hat{n}) d^2 \hat{n} \]

\[ = h_{\ell_1 \ell_2 \ell_3} \left( \frac{\ell_1}{m_1} \frac{\ell_2}{m_2} \frac{\ell_3}{m_3} \right) , \]

Triangle condition: \( \ell_1 \leq \ell_2 + \ell_3 \) for \( \ell_1 \geq \ell_2, \ell_3, \) +perms.

Parity condition: \( \ell_1 + \ell_2 + \ell_3 = 2n, \quad n \in \mathbb{N}, \)

Resolution: \( \ell_1, \ell_2, \ell_3 \leq \ell_{\text{max}}, \quad \ell_1, \ell_2, \ell_3 \in \mathbb{N}. \)
Bispectrum shapes

Local

Equilateral

Orthog.

ISW-lensing
Optimal $f_{NL}$ bispectrum estimator

$$\hat{f}_{NL} = \frac{1}{N} \sum B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \left[ (C^{-1}a)_{\ell_1}^{m_1} (C^{-1}a)_{\ell_2}^{m_2} (C^{-1}a)_{\ell_3}^{m_3} - 3C^{-1}_{\ell_1 m_1 \ell_2 m_2} (C^{-1}a)_{\ell_3}^{m_3} \right]$$

The theoretical template needs to be written in separable form. This can be done in different ways and alternative implementations differ basically in terms of the separation technique adopted and of the projection domain.

- **KSW** (Komatsu, Spergel & Wandelt 2003) separable template fitting + Skew-$C_l$ extension (Munshi & Heavens 2010)

- **Binned bispectrum** (Bucher, Van Tent & Carvalho 2009)

- **Modal expansion** (Fergusson, Liguori & Shellard 2009)
Going beyond the standard approach

• in Verde, Jimenez, Alvarez-Gaume, Heavens & Matarrese 2013 we provide an exact expression for the multi-variate joint probability distribution function of non-Gaussian fields primordially arising from local transformations of a Gaussian field.

• We apply our expression to the non-Gaussianity estimation from CMB maps and the halo mass function where we obtain analytical expressions.

• We also provide analytic approximations and their range of validity. and for the CMB we give a fast way to compute the PDF which is valid up to more than 7σ for $f_{NL}$ values not ruled out by current observations, which consists of expressing the PDF as a combination of bispectrum and trispectrum of the temperature maps.

• The resulting expression is valid for any kind of non-Gaussianity and is not limited to the local type.

These results may serve as the basis for a fully Bayesian analysis of the non-Gaussianity parameter.
Going to higher order?

Verde, Jimenez, Alvarez-Gaume, Heavens & Matarrese 2013

\[
\mathcal{P}(a \mid f_{NL}) = \frac{(\det C^{-1})^{1/2}}{(2\pi)^{n/2}} \exp \left[ -\frac{1}{2} \sum_{\ell \ell' mm'} a_{\ell}^* m (C^{-1})_{\ell m \ell' m'} a_{\ell'}^{m'} \right] \times \\
\left\{ 1 + \frac{1}{6} \sum_{\text{all } \ell; m_j} \langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \rangle \left[ (C^{-1} a)^{m_1}_{\ell_1} (C^{-1} a)^{m_2}_{\ell_2} (C^{-1} a)^{m_3}_{\ell_3} - 3(C^{-1})_{\ell_1 \ell_2 \ell_3} (C^{-1} a)^{m_3}_{\ell_3} \right] + \\
\frac{1}{24} \sum_{\text{all } \ell m} \langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} a_{\ell_4}^{m_4} \rangle \left[ 3(C^{-1})_{\ell_1 \ell_2 \ell_3} (C^{-1})_{\ell_2 \ell_3}^{m_3} \right] + \\
-6(C^{-1})_{\ell_1 \ell_2 \ell_3} (C^{-1} a)^{m_3}_{\ell_3} (C^{-1} a)^{m_4}_{\ell_4} + (C^{-1} a)^{m_1}_{\ell_1} (C^{-1} a)^{m_2}_{\ell_2} (C^{-1} a)^{m_3}_{\ell_3} (C^{-1} a)^{m_4}_{\ell_4} \right] + \\
\frac{1}{72} \sum_{l_1, \ldots, l_6} \langle a_{l_1}^{m_1} a_{l_2}^{m_2} a_{l_3}^{m_3} \rangle \langle a_{l_4}^{m_4} a_{l_5}^{m_5} a_{l_6}^{m_6} \rangle \left[ (C^{-1} a)^{m_1}_{l_1} (C^{-1} a)^{m_2}_{l_2} (C^{-1} a)^{m_3}_{l_3} (C^{-1} a)^{m_4}_{l_4} (C^{-1} a)^{m_5}_{l_5} (C^{-1} a)^{m_6}_{l_6} \right. \\
\left. -15(C^{-1})_{l_1 l_2 l_3} (C^{-1} a)^{m_3}_{l_3} (C^{-1} a)^{m_4}_{l_4} (C^{-1} a)^{m_5}_{l_5} (C^{-1} a)^{m_6}_{l_6} \right. \\
-15(C^{-1})_{l_1 l_2 l_3 l_4} (C^{-1} a)^{m_3 m_4}_{l_3 l_4} (C^{-1} a)^{m_5 m_6}_{l_5 l_6} + 45(C^{-1})_{l_1 l_2 l_3 l_4} (C^{-1} a)^{m_3 m_4}_{l_3 l_4} (C^{-1} a)^{m_5 m_6}_{l_5 l_6} \right\}
\]

It may become important if we want to detect NG in observables where $f_{NL}$ is large (e.g. in high-redshift probes) and/or if both $f_{NL}$ (→ leading order bispectrum) and $g_{NL}$ (leading-order trispectrum) are both depending on the same underlying physical coupling constant that we aim at determining.
The *Planck* modal bispectrum

Full 3D CMB bispectrum recovered from the *Planck* foreground-cleaned maps, including SMICA, NILC and SEVEM, using hybrid Fourier mode coefficients. These are plotted in three-dimensions with multipole coordinates \((l_1, l_2, l_3)\) on the tetrahedral domain out to \(l_{\text{max}} = 2000\). Several density contours are plotted with red positive and blue negative. The bispectra extracted from the different foreground-separated maps are almost indistinguishable.
Fundamental shapes (KSW)

- Results for the $f_{\text{NL}}$ parameters of the primordial local, equilateral, and orthogonal shapes, determined by the KSW estimator from the SMICA foreground-cleaned map. Both independent single-shape results and results marginalized over the point-source bispectrum and with the ISW-lensing bias subtracted are reported; error bars are 68% CL.

<table>
<thead>
<tr>
<th>SMICA</th>
<th>Independent</th>
<th>ISW-lensing subtracted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KSW</td>
<td>KSW</td>
</tr>
<tr>
<td>Local</td>
<td>9.8 ± 5.8</td>
<td>2.7 ± 5.8</td>
</tr>
<tr>
<td>Equilateral</td>
<td>−37 ± 75</td>
<td>−42 ± 75</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>−46 ± 39</td>
<td>−25 ± 39</td>
</tr>
</tbody>
</table>

- Union Mask U73 (73% sky coverage) used throughout. Diffusive inpainting pre-filtering procedure applied.
The coupling between weak lensing and Integrated Sachs-Wolfe (ISW) effects is the leading contamination to local NG. We have detected the ISW lensing bispectrum with a significance of 2.6 $\sigma$.

<table>
<thead>
<tr>
<th></th>
<th>SMICA</th>
<th>NILC</th>
<th>SEVEM</th>
<th>C-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>KSW</td>
<td>0.81 $\pm$ 0.31</td>
<td>0.85 $\pm$ 0.32</td>
<td>0.68 $\pm$ 0.32</td>
<td>0.75 $\pm$ 0.32</td>
</tr>
<tr>
<td>Binned</td>
<td>0.91 $\pm$ 0.37</td>
<td>1.03 $\pm$ 0.37</td>
<td>0.83 $\pm$ 0.39</td>
<td>0.80 $\pm$ 0.40</td>
</tr>
<tr>
<td>Modal</td>
<td>0.77 $\pm$ 0.37</td>
<td>0.93 $\pm$ 0.37</td>
<td>0.60 $\pm$ 0.37</td>
<td>0.68 $\pm$ 0.39</td>
</tr>
</tbody>
</table>

Results for the amplitude of the ISW-lensing bispectrum from the SMICA, NILC, SEVEM, and C-R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL.

<table>
<thead>
<tr>
<th></th>
<th>SMICA</th>
<th>NILC</th>
<th>SEVEM</th>
<th>C-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>7.1</td>
<td>7.0</td>
<td>7.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Equilateral</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>-22</td>
<td>-21</td>
<td>-21</td>
<td>-19</td>
</tr>
</tbody>
</table>

The bias in the three primordial $f_{NL}$ parameters due to the ISW-lensing signal for the 4 component-separation methods.
Standard inflation vs. NG

Standard inflation i.e.

- single scalar field
- canonical kinetic term
- slow-roll dynamics
- Bunch-Davies initial vacuum state
- standard Einstein gravity

predicts $O(10^{-2})$ primordial NG signal

→ no (presently) detectable primordial NG
Non-standard shapes: excited initial states

Non-Bunch-Davies vacua from trans-Planckian effects or features
Five exemplar flattened models constrained
NBD case

<table>
<thead>
<tr>
<th>Flattened model (Eq. number)</th>
<th>Raw $f_{NL}$</th>
<th>Clean $f_{NL}$</th>
<th>$\Delta f_{NL}$</th>
<th>$\sigma$</th>
<th>Clean $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat model (13)</td>
<td>70</td>
<td>37</td>
<td>77</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Non-Bunch-Davies (NBD)</td>
<td>178</td>
<td>155</td>
<td>78</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>Single-field NBD1 flattened (14)</td>
<td>31</td>
<td>19</td>
<td>13</td>
<td>2.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Single-field NBD2 squeezed (14)</td>
<td>0.8</td>
<td>0.2</td>
<td>0.4</td>
<td>1.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Non-canonical NBD3 (15)</td>
<td>13</td>
<td>9.6</td>
<td>9.7</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Vector model $L = 1$ (19)</td>
<td>−18</td>
<td>−4.6</td>
<td>47</td>
<td>−0.4</td>
<td>−0.1</td>
</tr>
<tr>
<td>Vector model $L = 2$ (19)</td>
<td>2.8</td>
<td>−0.4</td>
<td>2.9</td>
<td>1.0</td>
<td>−0.1</td>
</tr>
</tbody>
</table>

Systematic study of look-elsewhere effect is ongoing!
Planck results

• We detected the ISW-lensing bispectrum at the level expected in $\Lambda$CDM.

• We constrained early-Universe scenarios that generate primordial NG, including single-field inflation models, excited initial states, and direction-dependent vector models.

• We provided an initial survey of scale-dependent feature/resonance models, bounds on general single-field and multi-field model parameters: speed of sound, $c_s \geq 0.02$ (95% CL), in EFT parametrization ($c_s \geq 0.07$ for DBI inflation), curvaton decay fraction $r_D \geq 0.15$ (95% CL).

• We constrained the amplitude of the 4-point function in local model $\tau_{NL} < 2800$ (95% CL), using large-scale modulation of small-scale power-spectrum (Hanson & Lewis 2009).
Planck results

- The **simplest** inflation models (single-field slow-roll, standard kinetic term, BD initial vacuum state) are favoured by *Planck* data.

- Multi-field models are not ruled out but also not detected.

- Ekpyrotic/cyclic models (*the only alternative to inflation!* ) are either ruled out or under severe pressure.

- *Taken together, these constraints represent the highest precision tests to date of physical mechanisms for the origin of cosmic structure.*

- *Analyzing primordial NG proved an incredibly powerful tool to test fundamental physics at the highest achievable physical scales (10^{16} GeV?)*
Goals for Planck 2014: Full mission temperature data + (E-mode) polarization

Reduce error bars reaching an improvement
~ 40% with polarization + full mission data)

Constrain isocurvature NG models

Improve constraints on feature models, direction-dependent NG and constrain tensor NG

Constrain \( g_{\text{NL}} \) (for several shapes) and improve constraint on \( \tau_{\text{NL}} \)
Non-Gaussianity & Large-Scale Structure (LSS) of the Universe

(= primordial NG + NG from gravitational instability)
NG effects in LSS (mass)

- Bartolo, Matarrese & Riotto (2005) computed the effects of NG in the dark matter density fluctuations in a matter-dominated universe. Only for high values of $f_{NL}$ ($\sim 10$) the standard parameterization is valid. For smaller primordial NG strength non-Newtonian gravitational terms shift $f_{NL}$ by a term $\sim -1.6$ (see Verde & Matarrese 2010).

- Sefusatti & Komatsu (2007) show that LSS becomes competitive with CMB at $z > 2$.

- *but .. mass NG is not (all) what we measure with galaxy NG*
Searching for primordial NG with rare events (→ galaxies, etc.)

• Besides using standard statistical estimators, like (mass) bispectrum, trispectrum, three and four-point function, skewness, etc. ..., one can look at the tails of the distribution, i.e. at rare events.

• Rare events have the advantage that they often maximize deviations from what predicted by a Gaussian distribution, but have the obvious disadvantage of being rare! But remember that, according to Press-Schechter-like schemes, all collapsed DM halos correspond to (rare) peaks of the underlying density field.

• Matarrese, Verde & Jimenez (2000) and Verde, Jimenez, Kamionkowski & Matarrese showed that clusters at high redshift (z>1) can probe NG down to $f_{NL} \sim 10^2$


• Excellent agreement of analytical formulae with N-body simulations found by Grossi et al. 2009 ... and many others afterwards.
Dark matter halo clustering as (the most stringent?) constraint on NG

\[ \delta_{\text{halo}} = b \delta_{\text{matter}} \]

Dalal et al. (2007) have shown that halo bias is sensitive to primordial non-Gaussianity through a scale-dependent correction term

\[ \Delta b(k)/b \propto 2 f_{\text{NL}} \delta_c / k^2 \]

This opens interesting prospects for constraining or measuring NG in LSS but demands for an accurate evaluation of the effects of (general) NG on halo biasing.
Clustering of peaks (DM halos) of NG density field


\[ \xi_{h,M}(|x_1 - x_2|) = -1 + \exp \left\{ \sum_{N=2}^{\infty} \sum_{j=1}^{N-1} \frac{\mu^N \sigma^N}{j!(N-1)!} \frac{\zeta(N)}{j\times(N-j)} \right\} \]

(require use of path-integral, cluster expansion, multinomial theorem and asymptotic expansion). The analysis of NG models was motivated by a paper by Vittorio, Juszkiewicz and Davis (1986) on bulk flows.
Halo bias in NG models

• Matarrese & Verde 2008 applied this relation to the case of NG of the gravitational potential, obtaining the power-spectrum of dark matter halos modeled as high “peaks” (up-crossing regions) of height $\nu = \delta_c / \sigma_R$ of the underlying mass density field (Kaiser’s model). Here $\delta_c(z)$ is the critical overdensity for collapse (at redshift $a$) and $\sigma_R$ is the $rms$ mass fluctuation on scale $R$ ($M \sim R^3$).

• Account for motion of peaks (going from Lagrangian to Eulerian space), which implies (Catelan et al. 1998)

$$1 + \delta_h(x_{\text{Eulerian}}) = (1 + \delta_h(x_{\text{Lagrangian}}))(1 + \delta_R(x_{\text{Eulerian}}))$$

and (to linear order) $b=1+b_L$ (Mo & White 1996) to get the scale-dependent halo bias in the presence of NG initial conditions. *Corrections may arise from second-order bias and GR terms.*

• Alternative approaches (e.g. based on 1-loop calculations) by Taruya et al. 2008; Matsubara 2009; Jeong & Komatsu 2009. Giannantonio & Porciani 2010 improve fit to N-body simulations by assuming dependence on gravitational potential) $\rightarrow$ extension to bispectrum by Baldauf et al. 2011
Halo bias in NG models

• Extension to general (scale and configuration dependent) NG is straightforward

• In full generality write the $\phi$ bispectrum as $B_\phi(k_1,k_2,k_3)$. The relative NG correction to the halo bias is (Matarrese & Verde 2008)

$$\frac{\Delta b_h}{b_h} = \frac{\Delta_c(z)}{D(z)} \frac{1}{8\pi^2 \sigma_R^2} \int dk_1 k_1^2 M_R(k_1) \times$$

$$\int_{-1}^{1} d\mu M_R(\sqrt{\alpha}) \frac{B_\phi(k_1,\sqrt{\alpha},k)}{P_\phi(k)} \times \frac{1}{M_R(k)}$$

$$\alpha = k_1^2 + k_2^2 + 2k_1k_2\mu$$

• It also applies to non-local (e.g. “equilateral”) NG (DBI, ghost inflation, etc..) and universal NG term!! (→ see also Schmidt & Kamionkowski 2010).

• Calibrated to N-body simulations by Grossi et al. (2009), Desjacques et al. 2009; Pillepich et al. 2009; ...
### Observational status

<table>
<thead>
<tr>
<th>Data/method</th>
<th>$f_{NL}$ (local-type 95% CL)</th>
<th>reference</th>
<th>ad ref to bibl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photometric LRG - bias</td>
<td>$63^{+54}<em>{-35} - 101^{+101}</em>{-331}$</td>
<td>Slosar et al. 2008</td>
<td></td>
</tr>
<tr>
<td>Spectroscopic LRG- bias</td>
<td>$70^{+74}<em>{-38} - 139^{+139}</em>{-101}$</td>
<td>Slosar et al. 2008</td>
<td></td>
</tr>
<tr>
<td>QSO - bias</td>
<td>$8^{+26}<em>{-18} - 47^{+47}</em>{-37}$</td>
<td>Slosar et al. 2008</td>
<td></td>
</tr>
<tr>
<td>combined</td>
<td>$28^{+23}<em>{-19} - 42^{+42}</em>{-24}$</td>
<td>Slosar et al. 2008</td>
<td></td>
</tr>
<tr>
<td>NVSS–ISW</td>
<td>$105^{+64}<em>{-33} - 755^{+755}</em>{-1157}$</td>
<td>Slosar et al. 2008</td>
<td></td>
</tr>
<tr>
<td>NVSS–ISW</td>
<td>$236 \pm 127(2 - \sigma)$</td>
<td>Afshordi &amp; Tolley 2008</td>
<td></td>
</tr>
</tbody>
</table>

Xia et al. 2011 find $f_{NL} = 48 \pm 20$, $f_{NL} = 50 \pm 265$ and $f_{NL} = 183 \pm 95$ at 68% CL for local, equilateral and flat shape, respectively, using the NRAO VLA Sky Survey (NVSS), SDSS DR6 QSOs and the MegaZ-LRG (DR7)

Giannantonio et al 2013 find - $37 < f_{NL} < 25$ at 95% not including NVSS auto-correlation function in their analysis

Karagiannis, Shanks & Ross (2014) find $46 < f_{NL} < 158$ (local) at 95% CL using 22361 quasars from SDSS-III BOSS DR9

Leistedt, Peiris & Roth find - $49 < f_{NL} < 31$ (local) at 95% CL using a sample of 800,000 photometric quasars
# Observational prospects

<table>
<thead>
<tr>
<th>Data/method</th>
<th>$\Delta f_{NL} \times (1 - \sigma)$</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>BOSS–bias</td>
<td>18</td>
<td>Carbone et al 2008</td>
</tr>
<tr>
<td>ADEPT/Euclid–bias</td>
<td>1.5</td>
<td>Carbone et al 2008</td>
</tr>
<tr>
<td>PANNStarrs –bias</td>
<td>3.5</td>
<td>Carbone et al 2008</td>
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<tr>
<td>LSST–bias</td>
<td>0.7</td>
<td>Carbone et al 2008</td>
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<td>LSST-ISW</td>
<td>7</td>
<td>Afshordi&amp;Tolley 2008</td>
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<td>BOSS–bispectrum</td>
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<td>Sefusatti &amp; Komatsu 2008</td>
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<td>ADEPT/Euclid–bispectrum</td>
<td>3.6</td>
<td>Sefusatti &amp; Komatsu 2008</td>
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<td>Planck–Bispectrum</td>
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<td>Yadav et al. 2007</td>
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<td>BPOL–Bispectrum</td>
<td>2</td>
<td>Yadav et al. 2007</td>
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from: Carbone, Verde & Matarrese 2008
Future prospects on Non-Gaussianity search
Short term goals

• Improve $f_{\text{NL}}$ limits from CMB (*Planck*) with polarization & full data
• Look for more non-Gaussian shapes, scale-dependence, etc. ...
• Make use of bispectrum in 3D data
• Improve constraints on $g_{\text{NL}}$

Long term goals

• reconstruct inflationary action
• if (quadratic) NG turns out to be small for all shapes go on and search for $f_{\text{NL}} \sim 1$ non-linear GR effects and second-order radiation transfer function contributions
• what about intrinsic ($f_{\text{NL}} \sim 10^{-2}$) NG of standard inflation? CMB polarization + LSS + 21cm background + CMB spectral distortions
Contrary to earlier naive expectations, some level of non-Gaussianity is generically present in all inflation models. The level of non-Gaussianity predicted in the simplest (single-field, slow-roll, canonical kinetic term, BD initial state) inflation is below the minimum value detectable by Planck. \textit{GR effects} (implying $f_{NL}=-5/3$ in the squeezed limit) detectable by future galaxy surveys (Verde & Matarrese 2009).

Constraining/detecting non-Gaussianity is a powerful tool to discriminate among competing scenarios for perturbation generation (\textit{standard slow-roll inflation, curvaton, modulated-reheating, DBI, ghost inflation, multi-field, etc. ...}) some of which imply large non-Gaussianity. Thanks to the analysis of Planck data, non-Gaussianity has become the \textit{smoking-gun} for non-standard inflation models and a powerful tool to probe fundamental physics and the highest energy scales.

Primordial non-Gaussianity appears in a surprisingly large variety of cosmic phenomena, hence opening the possibility to constraining it by several complementary techniques.