The value of H_0 in the Inhomogeneous Universe

Giovanni Marozzi

University of Geneva

FFP14, Cosmology Conference,
Marseille, 16 July 2014

Based on:
I. Ben-Dayan, R. Durrer, GM, D. J. Schwarz, PRL 112, 221301 (2014).

Hubble constant H_0: local vs global

In a homogeneous and isotropic Universe H_0 is defined globally.

But the observed Universe is inhomogeneous and anisotropic on small scales (galaxy clusters and voids).

Local expansion rate, measured by cepheids and supernovae, does not necessarily agree with global fits to data from CMB anisotropies.

Local measurement: $H_0 = (73.8 \pm 2.4)\ \mathrm{km\ s^{-1} Mpc^{-1}}$
(Riess et al. (2011))

Global fit: $H_0 = (67.3 \pm 1.2)\ \mathrm{km\ s^{-1} Mpc^{-1}}$
(Planck Collaboration (2013))

Tension at 2.7σ level between local and CMB measurement.

Is this a problem? Is there new physics?

Sample and Cosmic Variance

Local measurements of H_0 are performed using the distance (modulus)-redshift relation of a sample of SNe.

The observed distance modulus μ is related to the bolometric flux Φ and the luminosity distance $d_{\rm L}$ by ($\log \equiv \log_{10}$)

$$\mu = -2.5 \log[\Phi/\Phi_{10\ \rm pc}] = 5 \log[d_{\rm L}/(10\ \rm pc)].$$

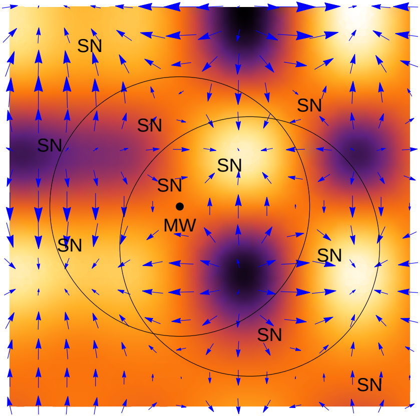
In a ΛCDM universe we then have

$$d_L(z) = \frac{1+z}{H_0/c} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + 1 - \Omega_m}}.$$

and $d_L(z) \simeq c[z + (1 - 3\Omega_m/4)z^2]/H_0$, for $z \leq 0.1$.

Even for arbitrarily precise measurements of fluxes and redshifts, the local H_0 differs from the global H_0 for two reason
a) Any SNe sample is finite (sample variance).
b) We observe only one realization of a random configuration of the local structure (cosmic variance).

Cosmic Variance



MW=Milky Way, SN=Supernova, dark region=under dense region, bright region=over dense region.
The arrows describe the velocity field of the SNe.
The two circles show how we would average over different samples of SN.

Cosmic Variance: previous literature

- In the context of Newtonian cosmology:

- Shi and Turner (1998), Wang, Spergel and Turner (1998), Buchert, Kerscher and Sicka (2000), Wojtak et al. (2013).

- In the context of a relativistic approach:

- Considering the ensemble variance of the expansion rate $\Theta = \nabla_\mu n^\mu$ averaged over a spatial volume (Li and Schwarz (2008), Wiegand and Schwarz (2012), Clarkson, Ananda and Larena (2009), Umeh, Larena and Clarkson (2011)).
- Modeling the local Universe with a "Swiss cheese" model (Fleury, Dupuy and Uzan (2013)).
- Modeling the local Universe with a "Hubble bubble" model and using the expansion rate (Marra, Amendola, Sawicki and Valkenburg (2013)).

Cosmic Variance: fully relativistic estimation

On the other hand, we have that

- Observers probe the past light-cone and not a spatial volume.
- The measured quantity is not an expansion rate, but a set of the bolometric fluxes and redshifts.

For the sake of generality, one should then not make any special hypothesis about how the fluctuations can be modeled around us, but consider only the cosmological standard model with stochastic inhomogeneities.

Keeping this in mind we want here give the first fully relativistic estimation of the effects of clustering on the error budget of local measurement of the Hubble parameter, taking light propagation effects fully into account.

We use cosmological perturbation theory with an almost scale-invariant initial power spectrum to determine the variance of the mean perturbation of the bolometric flux Φ from a standard candle.

Cosmic Variance

We then have $\sigma_\Phi^2 = \overline{\langle (\Phi_1/\Phi_0)^2 \rangle}$ with $\Phi = \Phi_0 + \Phi_1$ at first order in perturbation theory.
(Ben-Dayan, Gasperini, GM, Nugier, Veneziano (2012,2013))

For $z \ll 1$ the peculiar velocity terms dominates over the other contributions and we have:

$$\sigma_\Phi^2 \simeq 4 \left(\frac{1}{\mathcal{H}(z)\Delta\eta} \right)^2 \overline{\langle (\vec{v}_s \cdot \vec{n})^2 \rangle} \, .$$

In Fourier space and using the dimensionless power spectrum of the Bardeen potential today, $\mathcal{P}_\psi(k) = (k^3/2\pi^2)|\Psi_k(\eta_0)|^2$ we have

$$\sigma_\Phi^2 \simeq 4 \left(\frac{1}{\mathcal{H}(z)\Delta\eta} \right)^2 \frac{\tau^2(z)}{3} \int_{H_0}^{k_{\rm UV}} \frac{dk}{k} k^2 \mathcal{P}_\psi(k),$$

where $\tau(z) = \int_{\eta_{in}}^{\eta_s} d\eta \frac{a(\eta)}{a(\eta_s)} \frac{g(\eta)}{g(\eta_0)}$.

Hereafter, we use cosmological parameters from Planck, linear transfer function from Eisenstein and Hu (1998), and $k_{\rm UV} = 0.1 \, h \, \mathrm{Mpc}^{-1}$.

Cosmic Variance from a given SNe sample

To determine the dispersion of H_0 from a sample of SNe we use $H_0^2 \simeq c^2 z^2 / d_L^2$ for $z \ll 1$.

H_0 inferred from the observation of a single SN at redshift $z \ll 1$, is then expected to deviate from the true H_0 by

$$(\Delta H_0)^2 = \frac{H_0^2}{4} \overline{\langle (\Phi_1/\Phi_0)^2 \rangle}$$

In practice, observers do not have at their disposal many SNe at the same redshift, so the average over a sphere cannot be performed.

The (ensemble) variance of a locally measured Hubble parameter H_0 is then estimated by the covariance matrix of the fluxes of the sample of N observed SNe at positions $(z_i, \vec{n}_i)$

$$\left(\frac{\Delta H_0}{H_0}\right)^2 = \frac{1}{4N^2} \sum_{ij} \overline{\frac{\Phi_1(z_i, \vec{n}_i)}{\Phi_0(z_i)} \frac{\Phi_1(z_j, \vec{n}_j)}{\Phi_0(z_j)}}$$

Cosmic Variance from a given SNe sample

Using the Fourier representation of $\Phi_1(z_i, \vec{n}_i) = 2/(\mathcal{H}(z_i)\Delta\eta_i)\vec{v}_s(\vec{k}) \cdot \vec{n}_i$ we then have

$$\left(\frac{\Delta H_0}{H_0}\right)^2 = \frac{1}{N^2} \sum_{ij} \frac{V_{ij}}{\mathcal{H}(z_i)\Delta\eta_i \mathcal{H}(z_j)\Delta\eta_j} ,$$

with

$$V_{ij} = \tau(z_i)\tau(z_j) \int_{H_0}^{k_{\rm UV}} \frac{dk}{k} k^2 \mathcal{P}_\psi(k) I(k\Delta\eta_j, k\Delta\eta_i, (\vec{n}_i \cdot \vec{n}_j)),$$

and

$$\begin{aligned} I(x, y, \nu) &= \frac{1}{4\pi} \int d\Omega_{\hat{k}} e^{ix(\hat{k}\cdot\vec{n}_i)} e^{-iy(\hat{k}\cdot\vec{n}_j)} (\hat{k} \cdot \vec{n}_j)(\hat{k} \cdot \vec{n}_i) \\ &= \frac{xy(1-\nu^2)}{R^2} j_2(R) + \frac{\nu}{3}\left[j_0(R) - 2j_2(R)\right], \end{aligned}$$

where $\nu = (\vec{n}_i \cdot \vec{n}_j)$ and $R = \sqrt{x^2 + y^2 - 2\nu xy} = kd$.

d is the comoving distance between the SNe at $(z_i, \vec{n}_i)$ and $(z_j, \vec{n}_j)$.

Coherent vs incoherent effects

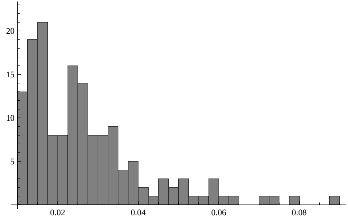
- If fluxes are perfectly coherent for all SNe $\Rightarrow$ $\overline{\Phi_1(z_i,\vec{n}_i)\Phi_1(z_j,\vec{n}_j)} = 4\sigma^2\Phi_0(z_j)\Phi_0(z_i)$, and $(\Delta H_0/H_0)^2 = \sigma^2$.
- If fluxes are perfectly incoherent for all SNe $\Rightarrow$ $\overline{\Phi_1(z_i,\vec{n}_i)\Phi_1(z_j,\vec{n}_j)} = \delta_{ij}4\sigma^2\Phi_0(z_j)\Phi_0(z_i)$, and $(\Delta H_0/H_0)^2 = \sigma^2/N$.

The reality lies somewhere in-between, wavelengths with $kd < 1$ being rather coherent while those with $kd > 1$ are rather incoherent.

To estimate the effect of the cosmic (co)variance for a realistic sample of SNe, we consider a sample of 155 SNe selected to lie in the range $0.01 \leq z \leq 0.1$ from the CfA3 and OLD samples (Hicken et al. (2009) and Jha, Riess and Kirshner (2007))

Cosmic Variance from CfA3+OLD sample

The redshift distribution of the CfA3+OLD sample is given by



We do not use the actual positions of the SNe on the sky, but we take two limit cases.

- Case A: Random distribution of the SNe directions over one hemisphere.
- Case B: All SNe inside a narrow cone ($\nu \simeq 1$).

The dispersion induced by inhomogeneities is then 2.2% (case A) and 3.3% (case B).
Using the H_0 value of Riess et al. (2011) we have

$$\Delta H_0 = (1.6 \div 2.4)\ \mathrm{km\ s^{-1} Mpc^{-1}}$$

H_0 local vs H_0 CMB

Considering the quoted observational error of 2.4 km/s/Mpc of Riess et al. (2011) and the induced cosmic variance we finally have

$$H_0 = [73.8 \pm 2.4 \pm (1.6 \div 2.4)] \ \mathrm{km\ s^{-1} Mpc^{-1}}.$$

The tension with the Planck measurement is reduced when taking this additional variance into account.

Adding the above errors in quadrature we obtain a deviation of 2.2 to 1.9 σ from $(H_0)_{\rm CMB}$! The concordance of the cosmological standard model is increased!!

This analysis is insensitive to smaller scales fluctuations for two reasons. The kernel $k^2\mathcal{P}_\psi(k)$ of the peculiar velocity contribution decreases at large k and small scale fluctuations are incoherent and their contribution to the variance decays like $1/N$, where N is the number of supernovae.

Cosmic Variance: arbitrarily large sample

What happen if we can observer an arbitrarily large sample with SNe distributed isotropically over directions?

In this case we can integrate $I(x, y, \nu)$ over all directions. With

$$\frac{1}{2}\int_{-1}^{1} d\nu I(x, y, \nu) = j_1(x) j_1(y)$$

and, for a normalized redshift distribution $\int dz\, s(z) = 1$, we have

$$\left(\frac{\Delta H_0}{H_0}\right)^2 = \int \frac{dk}{k} k^2 \mathcal{P}_\psi(k) \left(\int dz \tau(z) s(z) \frac{j_1(k\Delta\eta(z))}{\mathcal{H}(z)\Delta\eta(z)}\right)^2$$

Approximating the redshift distribution of the CfA3+OLD sample using an interpolating function and integrating from $z = 0.01$ to 0.1, we obtain a dispersion of about 1.8% which corresponds to an error of

$$\Delta H_0 = 1.3 \text{ km s}^{-1}\text{Mpc}^{-1} .$$

This is the minimal dispersion associated to a SN sample with the redshift space distribution given.

(Agrees with Wojtak et al. (2013), Li and Schwarz (2008) and Marra, Amendola, Sawicki and Valkenburg (2013)).

Cosmic Variance: redshift dependence

In the induced cosmic variance the errors from the nearby SNe, with small $\Delta\eta(z)$, give the largest contribution.

The dispersion can be reduced only by considering higher redshift SNe, but then the model dependence becomes more relevant.

For z close or larger than 0.3, we have to take into account also the other contributions to the perturbed flux

$$\frac{\Phi_1}{\Phi_0} = 2\left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right) v_{||s} + 2\psi_s - 2\left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right)\left(-\psi_s - 2\int_{\eta_s}^{\eta_o} d\eta' \partial_{\eta'} \psi(\eta')\right) - \frac{4}{\Delta\eta}\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \psi(\eta') + \frac{2}{\Delta\eta}\int_{\eta_s^{(0)}}^{\eta_o} d\eta' \frac{\eta' - \eta_s^{(0)}}{\eta_o - \eta'} \Delta_2 \psi(\eta')$$

In particular, the lensing term (in red) begins to dominate (Ben-Dayan, Gasperini, GM, Nugier, Veneziano (2013)).

Peculiar velocity field

In Riess et al. (2011) a partial reconstruction of the peculiar velocity fieldof SNe has been applied using the IRAS PSCz catalog (Branchini et al. (1999)).

This gives in the analysis of Riess et al. (2011) a global shift of H_0 at a sub percent level (Riess (private communication)).

Without take in consideration this partial reconstruction, a similar shift can been obtained considering the effect of inhomogeneities on the measured value of H_0 itself for a given sample (going to second order in perturbation theory and performing a light-cone average (Gasperini, GM, Nugier, Veneziano (2011)).

About the impact of the partial reconstruction of the peculiar velocity field on the induced cosmic variance, we have that the reconstruction comes from the density field in the neighborhood of the SNe and therefore contributes only to the incoherent (subleading) part of the theoretical induced cosmic variance in the large sample limit.

Conclusions

- We have estimated the impact of stochastic inhomogeneities on the error budget of H_0 local for a given sample of standard candles.
- We have obtained a general formula, which can be easily implemented and does not require an N-body simulation for each set of cosmological parameters.
- For typical sample with redshift range $0.01 < z < 0.1$ the induced error is not negligible but of the same order as the experimental error (2.2-3.3%).
- The cosmic variance is a fundamental barrier on the precision of a local measurement of H_0, and it reduces the tension with $(H_0)_{CMB}$.
- Even when the number of SNe is arbitrarily large, an irreducible error of about 1.8% remains due to cosmic variance of the local Universe.
- The cosmic variance can only be reduced by considering SNe with higher redshifts, but then the result becomes strongly dependent on other cosmological parameters like Ω_m and curvature.