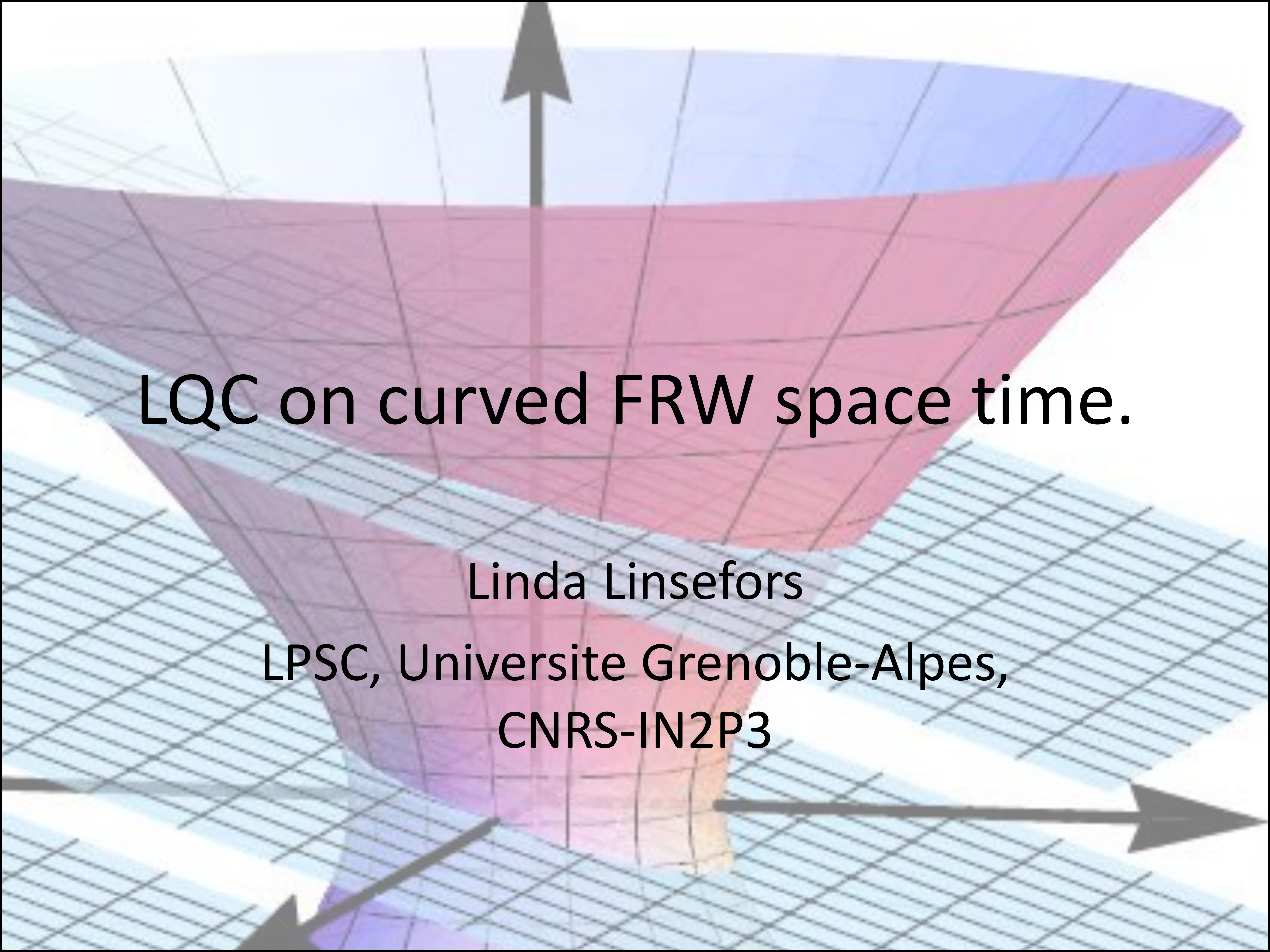

LQC on curved FRW space time.

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Friedmann–Robertson–Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\, r^2} + r^2 d\Omega^2 \right)$$

$$r' = r/b$$
$$a'(t) = b\, a(t)$$
$$k' = b^2\, k$$

$$H^2 = -\frac{k}{a^2} + \frac{\kappa}{3}\rho + \frac{\Lambda}{3}$$

$$H \;:=\; \dot{a}/a$$

$$\kappa := 8\pi G$$

$$H^2 = f\left(\frac{k}{a^2}, \text{matter}\right)$$

$$\Lambda = 3 f(0,0)$$

Friedmann Equation

$$0 = \mathcal{H}_{tot} := \mathcal{H}_G + \mathcal{H}_m$$

$$\mathcal{H}_m = V\rho$$

$$H^2 = f\left(\frac{k}{a^2}, \text{matter}\right) \quad \Rightarrow \quad H^2 = f\left(\frac{k}{a^2}, \rho\right)$$

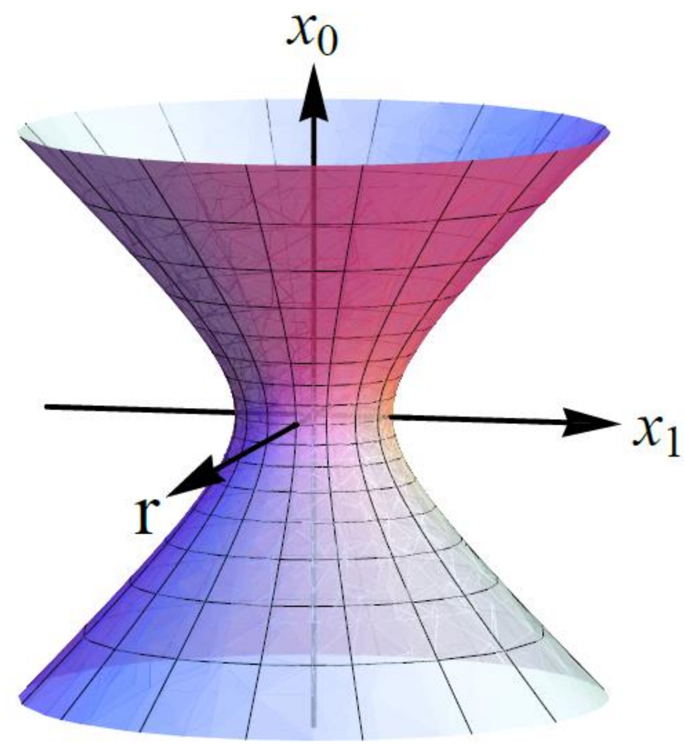
Case of constant energy density

$\rho = \rho_1 := \text{constant}$

$k = 0$

$H^2 = f\left(0, \rho_1\right)$

$\Lambda_{eff} = 3\ f\left(0, \rho_1\right)$



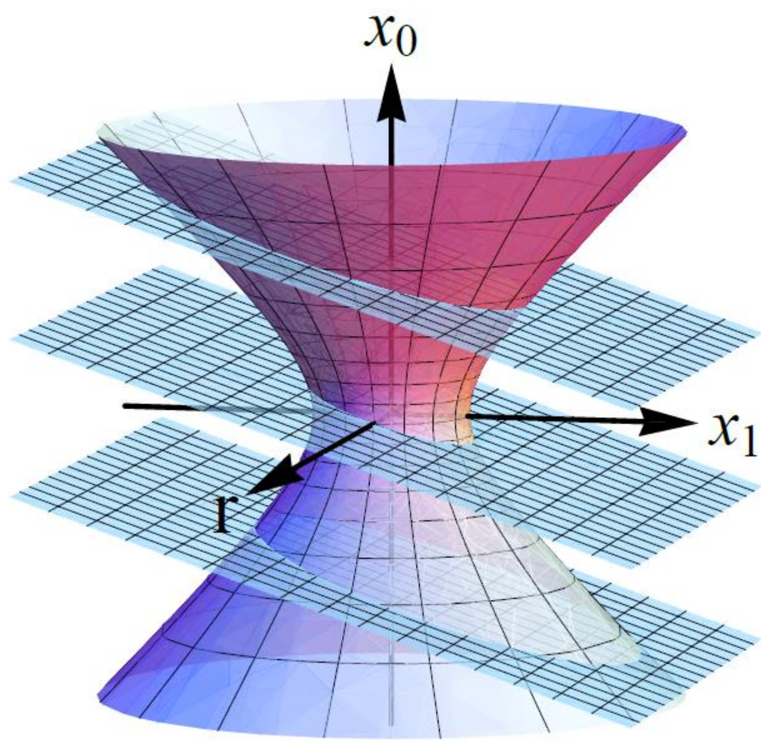
Case of constant energy density

$\rho = \rho_1 := \text{constant}$

~~$k = 0$~~

$$H^2 = f\,(0, \rho_1) - \frac{k}{a^2}$$

$$\Lambda_{eff} = 3\ f\,(0, \rho_1)$$



$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\,r^2} + r^2 d\Omega^2 \right)$$

LQC

$$H^2 = f\left(\frac{k}{a^2}, \rho\right) = f(0,\rho) - \frac{k}{a^2}$$

LQC $\Rightarrow$ $$f(0,\rho) = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$\Rightarrow$ $$f\left(\frac{k}{a^2}, \rho\right) = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) - \frac{k}{a^2}$$

Hamiltonian

$$V = vV_0 \qquad v := a^3 \qquad \{\alpha, v\} := \frac{1}{V_0}$$

$$0 = \mathcal{H}_G + \mathcal{H}_m = \mathcal{H}_G + V\rho \quad \Rightarrow \quad \rho = \frac{-\mathcal{H}_G}{vV_0}$$

$$H = \frac{\dot{v}}{3v} = \frac{1}{3v}\{v, \mathcal{H}_{tot}\} = \frac{1}{3vV_0}\frac{\partial(-\mathcal{H}_G)}{\partial\alpha}$$

Friedman equation: $\left(\frac{1}{3vV_0}\frac{\partial(-\mathcal{H}_G)}{\partial\alpha}\right)^2 = f\left(\frac{k}{v^{3/2}}, \frac{-\mathcal{H}_G}{vV_0}\right)$

$$\int d\alpha = \pm\int \frac{d(-\mathcal{H}_G)}{3vV_0\sqrt{f\left(\frac{k}{v^{2/3}}, \frac{-\mathcal{H}_G}{vV_0}\right)}}$$

Hamiltonian

$$V = vV_0 \qquad v := a^3 \qquad \{\alpha, v\} := \frac{1}{V_0}$$

$$\mathcal{H}_G = -vV_0 \frac{\rho_c}{2} \left(1 - \sqrt{1 - \frac{12}{\kappa\rho_c} \frac{k}{v^{2/3}}} \cos\left(\sqrt{\frac{3\kappa}{\rho_c}} [\alpha - \alpha_1(v)] \right) \right)$$

$$p = a^2 \qquad \{c, p\} = \frac{\kappa\gamma}{3V_0} \qquad \bar{\mu} := \frac{\lambda}{\sqrt{p}} \qquad \rho_c = \frac{3}{\kappa\gamma^2\lambda^2}$$

$$\mathcal{H}_G = -\frac{3V_0 p^{3/2}}{2\kappa\gamma^2\lambda^2} \left(1 - \sqrt{1 - \frac{12}{\kappa\rho_c} \frac{k}{p}} \cos\left(2\frac{\lambda}{\sqrt{p}} [c - c_1(p)] \right) \right)$$

Summery

ASUMPTIONS:

- Metric interpretation
- $\mathcal{H}_m = V\rho$ & any matter is allowed
- $H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$

RESULT:

- $H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) - \frac{k}{a^2}$
- $\mathcal{H}_G = -vV_0\frac{\rho_c}{2}\left(1 - \sqrt{1 - \frac{12}{\kappa\rho_c}\frac{k}{v^{2/3}}}\cos\left(\sqrt{\frac{3\kappa}{\rho_c}}[\alpha - \alpha_1(v)]\right)\right)$