Quantum formalism for systems with temporally varying discretization

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FFP14 @ Marseille
July 18th, 2014

based on PH arXiv:1401.6062, 1401.7731 and to appear
and B. Dittrich, PH, T. Jacobson wip
Discretization changing dynamics

Discrete gravity models and lattice field theory (subject to coarse graining/refining dynamics) generically feature temporally varying discretization

- interpret as dynamical coarse graining/refining operations
  
  see also Dittrich, Steinhaus '13

- leads to varying number of degrees of freedom in ‘time’

How to treat evolving lattice?

1. need ‘evolving’ phase and Hilbert spaces
2. unitarity?
3. observables?
4. constraints and symmetries?

**Goal**: understand this systematically!
Plan of the talk

1. Classical canonical dynamics

2. Quantum formalism

3. Vacuogenesis and QG dynamics

4. Summary and Outlook
Discretization changing dynamics: global moves

- **no Hamiltonian**: discrete evolution generated by time evolution moves
- **global time evolution moves**:
  1. correspond to space-time regions
  2. boundary hypersurfaces as discrete time steps
  3. evolve entire hypersurface at once
- discrete time evolution corresponds to gluing regions along common boundaries \( \Rightarrow \) evolves future boundary
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![Diagram showing time evolution with labeled steps and boundary hypersurfaces](image-url)
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![Diagram](image-url)
Associate to every region $R_k$ action $S_k(x_{k-1}, x_k)$
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Classical canonical dynamics [Marsden, West '01; Gambini, Pullin '03; Dittrich, PH '11,'13]

Associate to every region $R_k$ action $S_k(x_{k-1}, x_k)$
⇒ use as generating function

$(-p^0 := -\frac{\partial S_1(x_0, x_1)}{\partial x_0}, \quad +p^1 := \frac{\partial S_1(x_0, x_1)}{\partial x_1})$

$(-p$: pre–momenta, \quad $+p$: post–momenta)
Classical canonical dynamics

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$\Rightarrow$ use as generating function

$(\# \text{ of } x_0) \neq (\# \text{ of } x_1) \text{ allowed}$

$-p^0 := -\frac{\partial S_1(x_0, x_1)}{\partial x_0}$, $+p^1 := \frac{\partial S_1(x_0, x_1)}{\partial x_1}$

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defines time evolution map

$(x_0, -p^0) \mapsto (x_1, +p^1)$
Classical canonical dynamics \[\text{[Marsden, West '01; Gambini, Pullin '03; Dittrich, PH '11,'13]}\]

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\[-p^0 := -\frac{\partial S_1(x_0, x_1)}{\partial x_0}, \quad +p^1 := \frac{\partial S_1(x_0, x_1)}{\partial x_1}\]

\(-p: \text{ pre–momenta,} \quad +p: \text{ post–momenta}\)

● defines time evolution map

\((x_0, -p^0) \mapsto (x_1, +p^1)\)

● similarly, use \(S_2(x_1, x_2)\) as gen. fct.

\[-p^1 = -\frac{\partial S_2}{\partial x_1}\]

eom \(\frac{\partial S_1}{\partial x_1} + \frac{\partial S_2}{\partial x_1} = 0 \iff +p^1 = -p^1 \text{ momentum matching}\)
Constraints in the discrete  [Dittrich, PH ’11, ’13]

- evolution $0 \rightarrow 1$ defined by
  \[
  -p^0 := -\frac{\partial S_1(x_0, x_1)}{\partial x_0}, \quad +p^1 := \frac{\partial S_1(x_0, x_1)}{\partial x_1}
  \]

⇒ obtain two types of constraints if $\frac{\partial^2 S_1}{\partial x_0^i \partial x_1^j}$ has left and right null vectors
  - $+C^1(x_1, +p^1) = 0$ \Rightarrow post–constraints
  - $-C^0(x_0, -p^0) = 0$ \Rightarrow pre–constraints

- time evol. no longer unique:

  e.g., $-C^0(x_0, -p^0) = 0 \Rightarrow x_1 = x_1(x_0, -p^0, \lambda_1^m)$,

  $\lambda_1$: a priori free parameter
non-trivialities arise when gluing 2 regions: impose both $+C^1, -C^1$
generally, $+C^1 \neq -C^1$

$\{ -C_i^1, -C_j^1 \} \approx 0 \approx \{ +C_i^1, +C_j^1 \}$ but $\{ -C_i^1, +C_j^1 \} \neq 0$

possibilities at step 1:

1. $C^1 = -C^1 = +C^1 \Rightarrow 1\text{st class gauge symmetry generator}$

2. 2nd class $\Rightarrow$ fixes free parameters

3. $-C^1$ indep. of post–constraints but 1st class $\Rightarrow$ non-trivial coarse graining condition for data of move $0 \rightarrow 1$

4. $+C^1$ indep. of pre–constraints but 1st class $\Rightarrow$ non-trivial coarse graining condition for data of move $1 \rightarrow 2$
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possibilities at step 1:

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4. $+C^1$ indep. of pre–constraints but 1st class $\Rightarrow$ non-trivial coarse graining condition for data of move 1 $\rightarrow$ 2
Coarse graining dynamics and pre–constraints

P. Höhn (Perimeter)
Coarse graining dynamics and pre–constraints

\[
\tilde{S}_{02} := s_1 + s_2 \quad \mid_{x_1^{sol}}
\]

\[+ C^2 \]

\[- \tilde{C}^0 \]

new

⇒ constraints ‘propagate’ and become move/region dependent

⇒ propagation of information becomes move/region dependent!
restrict to configuration spaces $Q \simeq \mathbb{R}^{N_k}$

- impose constraints in quantum theory à la Dirac: $\hat{C}|\psi_{\text{phys}}\rangle = 0$
- quantum pre-/post–constraints:
  1. self-adjoint w.r.t. $\mathcal{H}_{\text{kin}}^k = L^2(\mathbb{R}^{N_k}, dx_k)$
  2. have absolutely cont. spectrum
  3. orbits non-compact

$\Rightarrow$ proceed by group averaging [Marolf '95, '00]:

- post–physical states: $\psi_{\text{phys}}^1 := + \mathbb{P}_1 \psi_{\text{kin}}^1 = \prod_I \delta(\hat{C}_I^1) \psi_{\text{kin}}^1$
- pre–physical states: $\psi_{\text{phys}}^0 := - \mathbb{P}_0 \psi_{\text{kin}}^0 = \prod_I \delta(-\hat{C}_I^0) \psi_{\text{kin}}^0$

$$\delta(\hat{C}) := \frac{1}{2\pi} \int ds \ e^{is\hat{C}}$$

- physical inner product on pre-/post–physical Hilbert spaces $\pm \mathcal{H}_{\text{phys}}^k$

$$\langle \pm \psi_{\text{phys}}^k | \pm \xi_{\text{phys}}^k \rangle_{\text{phys}} = \langle \psi_{\text{kin}}^k | \pm \mathbb{P}_k \xi_{\text{kin}}^k \rangle_{\text{kin}}$$
Quantum dynamics \cite{PH14}

\[
\mathcal{H}_0^{\text{kin}} \quad \text{Hohn (Perimeter)}
\]

Discretization changing dynamics
Quantum dynamics [PH ’14]

\[ \mathcal{H}_0 \xrightarrow{P_0 \rightarrow 1} \mathcal{H}_1 \]

\[ \mathcal{H}_0^{\text{kin}} \]

\[ \mathcal{H}_1^{\text{kin}} \]

\[ -\mathcal{H}_0^{\text{phys}} \]

\[ +\mathcal{H}_1^{\text{phys}} \]
Quantum dynamics [PH '14]

\[
\begin{align*}
\mathcal{H}_0^\text{kin} & \quad \mathcal{H}_1^\text{kin} \\
\mathcal{H}_0^\text{phys} & \quad \mathcal{H}_1^\text{phys}
\end{align*}
\]

\[
\begin{align*}
P_0 \rightarrow & \quad P_0 \\
- & \quad - \\
+ & \quad +
\end{align*}
\]

\[
\begin{align*}
P_1 \rightarrow & \quad P_1 \\
- & \quad - \\
+ & \quad +
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\]
Quantum dynamics [PH '14]

\[ H^{\text{kin}}_0 \to P_0 \to 0 \to P_0 \to H^{\text{phys}}_0 \]

\[ H^{\text{kin}}_1 \to P_1 \to 0 \to P_1 \to H^{\text{phys}}_1 \]

\[ -H^{\text{phys}}_0 \to U_{0\to1} \to +H^{\text{phys}}_1 \]

\[ -P_0 \to P_0 \to +P_1 \to +P_1 \]
propagator ansatz for evolution $0 \rightarrow 1$

$$K_{0\rightarrow 1}(x_0, x_1) = M_{0\rightarrow 1}(x_0, x_1) e^{iS_1(x_0, x_1)}$$

e.g. projector $P_{0\rightarrow 1}$

$$+\psi_1^{\text{phys}} = P_{0\rightarrow 1} \psi_0^{\text{kin}} = \int dx_0 K_{0\rightarrow 1} \psi_0^{\text{kin}}$$

$\Rightarrow$ requires $+\hat{C}^1 K_{0\rightarrow 1} = 0 = -\hat{C}^0(K_{0\rightarrow 1})^*$

$\Rightarrow$ $K_{0\rightarrow 1} = +\mathbb{P}_1 (-\mathbb{P}_0)^* \kappa_{0\rightarrow 1}(x_0, x_1)$, $\kappa_{0\rightarrow 1}$ kinematical prop.
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⇒ $K_{0 \rightarrow 1} = + P_1 ( - P_0)^* \kappa_{0 \rightarrow 1}(x_0, x_1), \quad \kappa_{0 \rightarrow 1} \text{ kinematical prop.}$

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Quantum dynamics [PH '14]

- Propagator ansatz for evolution \(0 \rightarrow 1\)

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K_{0 \rightarrow 1}(x_0, x_1) = M_{0 \rightarrow 1}(x_0, x_1) e^{iS_1(x_0, x_1)}
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- E.g. projector \(P_{0 \rightarrow 1}\)

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+\psi_{\text{phys}}^1 = P_{0 \rightarrow 1} \psi_{\text{kin}}^0 = \int dx_0 K_{0 \rightarrow 1} \psi_{\text{kin}}^0
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\(\Rightarrow\) Requires \(+\hat{C}_1 K_{0 \rightarrow 1} = 0 = -\hat{C}_0(K_{0 \rightarrow 1})^*\)

\(\Rightarrow\) \(K_{0 \rightarrow 1} = +\mathbb{P}_1 (-\mathbb{P}_0)^* \kappa_{0 \rightarrow 1}(x_0, x_1), \quad \kappa_{0 \rightarrow 1}\) [kinematical prop.]

\(\Rightarrow\) \(+\psi_{\text{phys}}^1 = U_{0 \rightarrow 1} -\psi_{\text{phys}}^0\)

- \(U_{0 \rightarrow 1}\) unitary: 

\[
\langle +\psi_{\text{phys}}^1 | +\xi_{\text{phys}}^1 \rangle_{\text{phys}} = \langle -\psi_{\text{phys}}^0 | -\xi_{\text{phys}}^0 \rangle_{\text{phys}}
\]
Composition of global moves [PH '14]

non-trivialities arise when gluing 2 regions

⇒ amounts to concatenation of propagators

\[ K_{0\rightarrow 2} = \int dx_1 \ K_{1\rightarrow 2} \ K_{0\rightarrow 1} \]
Composition of global moves \cite{PH14}

non-trivialities arise when gluing 2 regions

\[ \Sigma_2 \]

\[ \Sigma_1 \]

\[ \Sigma_0 \]

\[ R_2 \]

\[ R_1 \]

\[ \Rightarrow \text{ amounts to concatenation of propagators} \]

\[ K_{0 \rightarrow 2} = \int dx_1 \; ^{+}P_2 \; \kappa_{1 \rightarrow 2} \; ^{-}P_1 \; ^{+}P_1 \; \kappa_{0 \rightarrow 1} \; ^{-}P_0 \]
Composition of global moves [PH ’14]

non-trivialities arise when gluing 2 regions
- recall constraint classification at 1:
  1. $\hat{C}^1 = +\hat{C}^1 = -\hat{C}^1$: 1st class symmetry generators
  2. 2nd class $\Rightarrow$ solve classically
  3. $-\hat{C}_A^1$ 1st class, but indep. of $+\hat{C}^1$
  4. $+\hat{C}_B^1$ 1st class, but indep. of $-\hat{C}^1$

$\Rightarrow$ amounts to concatenation of propagators

$$K_{0\rightarrow 2} = \int dx_1 \, +p_2 \, \kappa_{1\rightarrow 2} -p_1 +p_1 \, \kappa_{0\rightarrow 1} -p_0$$
Composition of global moves \[\text{[PH '14]}\]

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\(\Rightarrow\) amounts to concatenation of propagators

\[
K_{0\to2} = \int dx_1 \, +\mathbb{P}_2 \, \kappa_{1\to2} \, -\mathbb{P}^A_1 \, (\mathbb{P}_1)^2 \, +\mathbb{P}^B_1 \, \kappa_{0\to1} \, -\mathbb{P}_0
\]
Composition of global moves

non-trivialities arise when gluing 2 regions

recall constraint classification at 1:

1. \( \hat{\mathcal{C}}^1 = + \hat{\mathcal{C}}^1 = - \hat{\mathcal{C}}^1 \): 1st class symmetry generators
2. 2nd class \( \Rightarrow \) solve classically
3. \( - \hat{\mathcal{C}}_A^1 \): 1st class, but indep. of \( + \hat{\mathcal{C}}^1 \)
4. \( + \hat{\mathcal{C}}_B^1 \): 1st class, but indep. of \( - \hat{\mathcal{C}}^1 \)

\( \Rightarrow \) amounts to concatenation of propagators

\[ K_{0 \rightarrow 2} = \int dx_1 \, +P_2 \kappa_{1 \rightarrow 2} - P^A_1 (P_1)^2 + P^B_1 \kappa_{0 \rightarrow 1} - P_0 \]

“\((P_1)^2 \rightarrow \infty\)” (integration over non-compact gauge orbit)
Composition of global moves \cite{PH14}

non-trivialities arise when gluing 2 regions

\begin{itemize}
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\end{itemize}

$\Rightarrow$ amounts to concatenation of propagators

$$K_{0\rightarrow2} = \int dx_1 +\mathbb{P}_2 \kappa_{1\rightarrow2} -\mathbb{P}^A_1 \mathbb{P}_1 +\mathbb{P}^B_1 \kappa_{0\rightarrow1} -\mathbb{P}_0$$

"$(\mathbb{P}_1)^2 \rightarrow \infty$" (integration over non-compact gauge orbit)

$\Rightarrow$ regularize by dropping one instance of $\mathbb{P}_1$
non-trivialities arise when gluing 2 regions

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$\Rightarrow$ amounts to concatenation of propagators

$$K_{0\to2} = \int dx_1 +P_2^{\kappa_{1\to2}} -P_A^1 P_1^1 +P_B^1 \kappa_{0\to1} -P_0^0$$

- new effective constraints $-\hat{C}^0$, $+\hat{C}^2$ also arise in QT
Composition of global moves [PH '14]

non-trivialities arise when gluing 2 regions

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$$K_{0\to2} = \int dx_1 \; +P_2 \kappa_{1\to2} -P_1^A P_1 +P_1^B \kappa_{0\to1} -P_0$$

- new effective constraints $-\tilde{C}^0$, $+\tilde{C}^2$ also arise in QT

$\Rightarrow$ non-unitary projections physical Hilbert spaces

$+\tilde{H}^\text{phys}_2 := +\tilde{P}_2 \left( +H^\text{phys}_2 \right)$ and $-\tilde{H}^\text{phys}_0 := -\tilde{P}_0 \left( -H^\text{phys}_0 \right)$

$\Rightarrow$ non-trivial dynamical coarse graining of discretization
non-trivialities arise when gluing 2 regions

- recall constraint classification at 1:
  1. $\hat{\mathcal{C}}^1 = + \hat{\mathcal{C}}^1 = - \hat{\mathcal{C}}^1$: 1st class symmetry generators
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$$K_{0\to2} = \int d\chi_1 +\mathbb{P}_2 \kappa_{1\to2} -\mathbb{P}_1^A \mathbb{P}_1 +\mathbb{P}_1^B \kappa_{0\to1} -\mathbb{P}_0$$

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$\Rightarrow$ non-unitary projections physical Hilbert spaces

$+\hat{\mathcal{H}}_2^{\text{phys}} := +\mathbb{P}_2 (+\mathcal{H}_2^{\text{phys}})$ and $-\hat{\mathcal{H}}_0^{\text{phys}} := -\mathbb{P}_0 (-\mathcal{H}_0^{\text{phys}})$

$\Rightarrow$ non-trivial dynamical coarse graining of discretization

$\Rightarrow$ physical Hilbert spaces associated to (boundary) of region (rather than time step) as in ‘general boundary formulation’ [Oeckl ’03, ’08]
Physical dofs on varying discretizations [Dittrich, PH '13; PH '14]

- Propagating dofs must commute with constraints, to be well-defined on $-\mathcal{H}_1^{\text{phys}} / +\mathcal{H}_1^{\text{phys}}$

1. Pre-observables $-\hat{O}^1$: $[-\hat{C}^i, -\hat{O}^1] = 0$

2. Post-observables $+\hat{O}^1$: $[+\hat{C}^i, +\hat{O}^1] = 0$

\[
\begin{align*}
\text{integrate} & \\
S_2 & : +\hat{O}^2(\hat{x}_2, +\hat{p}^2) & 2 \\
S_1 & : -\hat{O}^1(\hat{x}_1, -\hat{p}^1) & 1 \\
S_2 & : +\hat{O}^1(\hat{x}_1, +\hat{p}^1) & 1 \\
S_1 & : -\hat{O}^0(\hat{x}_0, -\hat{p}^0) & 0 \\
\end{align*}
\]
propagating dofs must commute with constraints, to be well-defined on $-\mathcal{H}_1^\text{phys}/+\mathcal{H}_1^\text{phys}$

1. pre–observables $-\hat{O}^1$: $[-\hat{C}_i^1, -\hat{O}^1] = 0$
2. post–observables $+\hat{O}^1$: $[+\hat{C}_i^1, +\hat{O}^1] = 0$

integrating out 1: $-\hat{O}^0/+\hat{O}^2$ must commute with new $-\hat{C}^0, +\hat{C}^2$

$\Rightarrow$ ‘too finely grained’ dofs do not commute with new constraints
$\Rightarrow$ dynamical coarse graining projects out dofs irreversibly
idea: newly added modes ‘Euclideanized’, born in vacuum

(almost) trivial toy model:
‘nothing’ \rightarrow scalar field on single vertex

\[ S_1 = \frac{1}{2} \phi_1^2 \quad \rightarrow \quad S_{\text{eucl}}^1 = \frac{1}{2} i \phi_1^2 \]

post–constraint as annihilation operator:

\[ i^+ \hat{C}_{\text{eucl}}^1 = \hat{a} = \hat{\phi}_1 + i \hat{\rho}_1 \]

\( \Rightarrow \) post–physical state is unique (Gaussian) vacuum state

\[ + \psi_1^{\text{phys}} \sim e^{-\frac{1}{2} \phi_1^2} \]

more complicated for large lattice
Discretization changing dynamics in QG [Dittrich, Steinhaus '13, PH '14]

**so far:**

states evolve in background time (e.g. lattice field theory)

**in QG:**

physical states ‘timeless’, do **not** evolve in external time

⇒ time evolution relational

⇒ discrete state evolution not ‘time evolution’

⇒ discretization changing dynamics = coarse graining/refining

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**3D**

- diffeo sym. perserved
- ‘evolution’ as change of representation

**4D**

- sym. broken
- non-unitary coarse graining
- refining non-hyperbolic

⇒ need ‘dynamical cylindrical consistency’ as in Dittrich '12, Dittrich, Steinhaus '13
**Summary and Outlook**

- systematic classical and quantum formalism for discretization changing dynamics available
- constraints, observables, Hilbert spaces, ... region dependent
- non-trivial coarse graining $\Rightarrow$ non-unitary projections of physical Hilbert spaces and observables
- analogously with Pachner move dynamics

**goal:** better understand

1. discretization changing dynamics in QG
2. ‘vacuogenesis’