Quantum formalism for systems with temporally varying discretization

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based on PH arXiv:1401.6062, 1401.7731 and to appear
and B. Dittrich, PH, T. Jacobson wip
(classical formalism B. Dittrich, PH arXiv:1303.4294, 1108.1974)

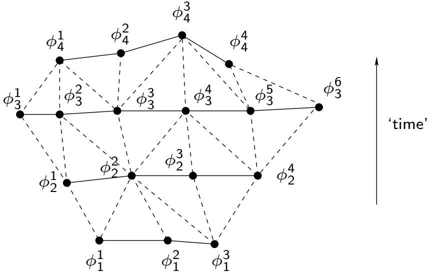
Discretization changing dynamics

Discrete gravity models and lattice field theory (subject to coarse graining/refining dynamics) generically feature temporally varying discretization

- interpret as dynamical coarse graining/refining operations

  see also Dittrich, Steinhaus '13

- leads to varying number of degrees of freedom in 'time'
- How to treat evolving lattice?
  1. need 'evolving' phase and Hilbert spaces
  2. unitarity?
  3. observables?
  4. constraints and symmetries?



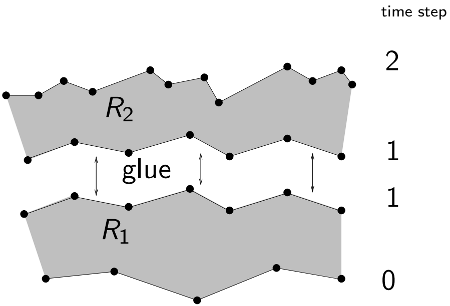
Goal: understand this systematically!

Plan of the talk

1. Classical canonical dynamics
2. Quantum formalism
3. Vacuogenesis and QG dynamics
4. Summary and Outlook

Discretization changing dynamics: global moves

- no Hamiltonian: discrete evolution generated by time evolution moves
- global time evolution moves:
  1. correspond to space-time regions
  2. boundary hypersurfaces as discrete time steps
  3. evolve entire hypersurface at once
- discrete time evolution corresponds to gluing regions along common boundaries ⇒ evolves future boundary



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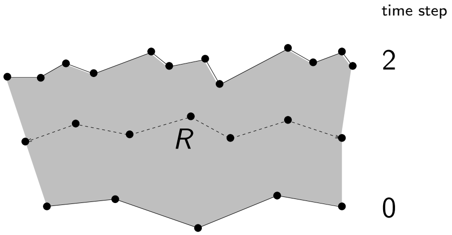

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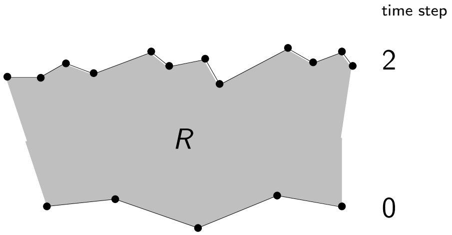
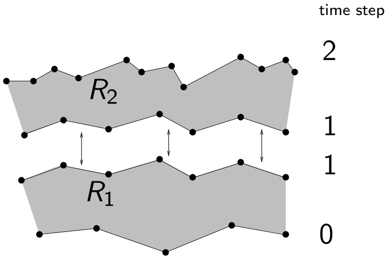


Associate to every region R_k action $S_k(x_{k-1}, x_k)$



Classical canonical dynamics [Marsden, West '01; Gambini, Pullin '03; Dittrich, PH '11,'13]

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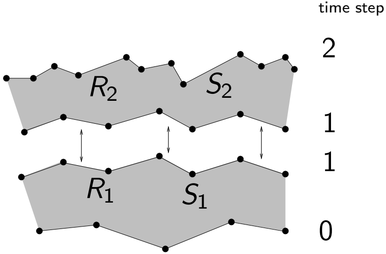
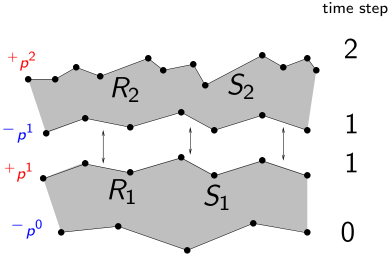


Associate to every region R_k action $S_k(x_{k-1}, x_k)$

$\Rightarrow$ use as generating function $(\#\text{ of } x_0) \neq (\#\text{ of } x_1)$ allowed

$$ {}^{-}p^0 := -\frac{\partial S_1(x_0, x_1)}{\partial x_0} \quad , \quad {}^{+}p^1 := \frac{\partial S_1(x_0, x_1)}{\partial x_1} $$

${}^{-}p$: pre–momenta, ${}^{+}p$: post–momenta



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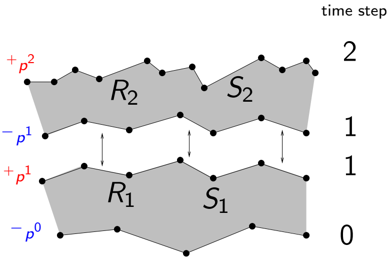

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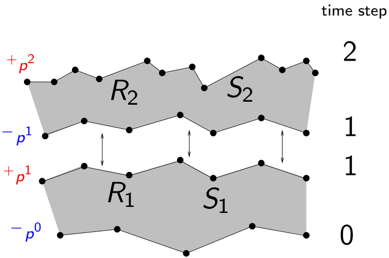
$$ (x_0, {}^{-}p^{0}) \mapsto (x_1, {}^{+}p^{1}) $$

- similarly, use $S_2(x_1, x_2)$ as gen. fct.

$$ {}^{-}p^{1} = -\frac{\partial S_2}{\partial x_1} $$



- eom $\frac{\partial S_1}{\partial x_1} + \frac{\partial S_2}{\partial x_1} = 0 \Leftrightarrow {}^{+}p^{1} = {}^{-}p^{1}$ *momentum matching*

Constraints in the discrete [Dittrich, PH '11, '13]

- evolution $0 \longrightarrow 1$ defined by

$$ {}^{-}p^{0} := -\frac{\partial S_1(x_0, x_1)}{\partial x_0} \quad , \quad {}^{+}p^{1} := \frac{\partial S_1(x_0, x_1)}{\partial x_1} $$

$\Rightarrow$ obtain two types of constraints if $\frac{\partial^2 S_1}{\partial x_0^i \partial x_1^j}$ has left and right null vectors

  - ${}^{+}C^{1}(x_1, {}^{+}p^{1}) = 0$ $\quad \Rightarrow$ post–constraints
  - ${}^{-}C^{0}(x_0, {}^{-}p^{0}) = 0$ $\quad \Rightarrow$ pre–constraints

- time evol. no longer unique:

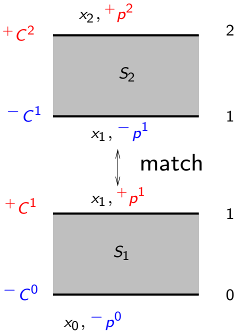
e.g., ${}^{-}C^{0}(x_0, {}^{-}p^{0}) = 0 \Rightarrow x_1 = x_1(x_0, {}^{-}p^{0}, \lambda_1^m)$,

λ_1: *a priori* free parameter

Constraint classification [Dittrich, PH '13, PH to appear]

- non-trivialities arise when gluing 2 regions: impose both ${}^{+}C^1, {}^{-}C^1$
  generally, ${}^{+}C^1 \neq {}^{-}C^1$
- $\{{}^{-}C^1_i, {}^{-}C^1_j\} \approx 0 \approx \{{}^{+}C^1_i, {}^{+}C^1_j\}$ but $\{{}^{-}C^1_i, {}^{+}C^1_j\} \neq 0$

- possibilities at step 1:
  1. $C^1 = {}^{-}C^1 = {}^{+}C^1 \Rightarrow$ 1st class gauge symmetry generator
  2. 2nd class $\Rightarrow$ fixes free parameters
  3. ${}^{-}C^1$ indep. of post–constraints but 1st class $\Rightarrow$ non-trivial coarse graining condition for data of move $0 \rightarrow 1$
  4. ${}^{+}C^1$ indep. of pre–constraints but 1st class $\Rightarrow$ non-trivial coarse graining condition for data of move $1 \rightarrow 2$



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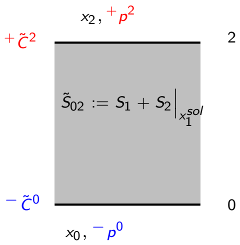
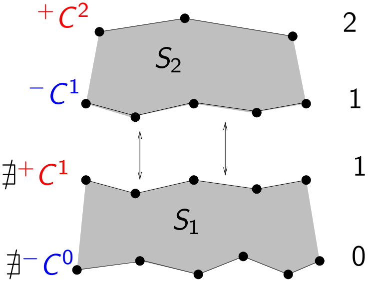
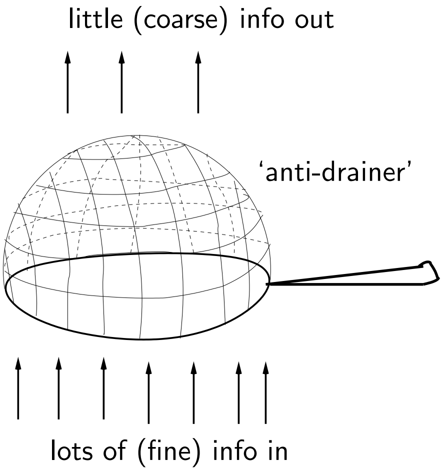


Coarse graining dynamics and pre–constraints





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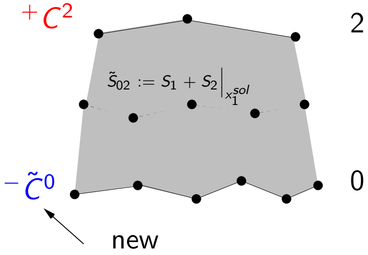
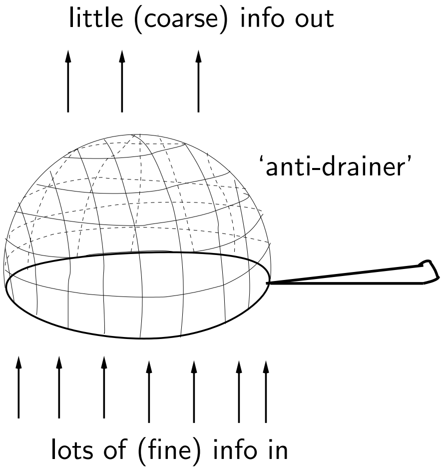




⇒ constraints 'propagate' and become move/region dependent
⇒ propagation of information becomes move/region dependent!

Quantization: general construction [PH '14]

restrict to configuration spaces $\mathcal{Q} \simeq \mathbb{R}^{N_k}$

- impose constraints in quantum theory á la Dirac: $\hat{C}|\psi^{\rm phys}\rangle = 0$
- quantum pre–/post–constraints:
  1. self-adjoint w.r.t. $\mathcal{H}_k^{\rm kin} = L^2(\mathbb{R}^{N_k}, dx_k)$
  2. have absolutely cont. spectrum
  3. orbits non-compact

$\Rightarrow$ proceed by group averaging [Marolf '95, '00]:

post–physical states: ${}^{+}\psi_1^{\rm phys} := {}^{+}\mathbb{P}_1\, \psi_1^{\rm kin} = \prod_I \delta({}^{+}\hat{C}_I^1)\, \psi_1^{\rm kin}$

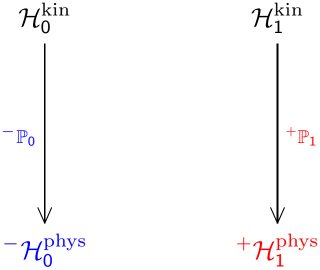
pre–physical states: ${}^{-}\psi_0^{\rm phys} := {}^{-}\mathbb{P}_0\, \psi_0^{\rm kin} = \prod_I \delta({}^{-}\hat{C}_I^0)\, \psi_0^{\rm kin}$

$$\delta(\hat{C}) := \frac{1}{2\pi} \int ds\, e^{is\hat{C}}$$

- physical inner product on pre–/post–physical Hilbert spaces ${}^{\pm}\mathcal{H}_k^{\rm phys}$

$\langle {}^{\pm}\psi_k^{\rm phys} \big| {}^{\pm}\xi_k^{\rm phys} \rangle_{\rm phys} = \langle \psi_k^{\rm kin} \big| {}^{\pm}\mathbb{P}_k \xi_k^{\rm kin} \rangle_{\rm kin}$

Quantum dynamics [PH '14]



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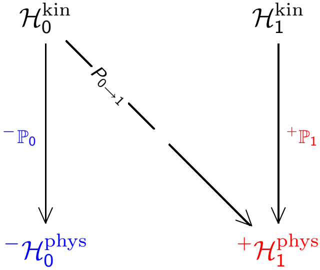

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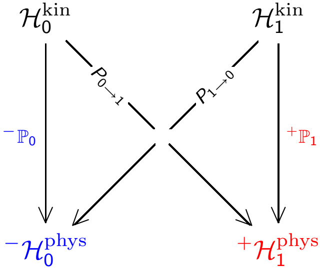

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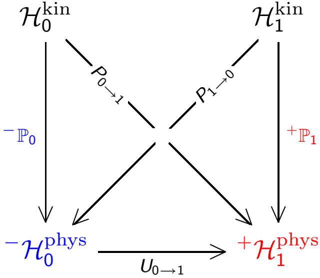
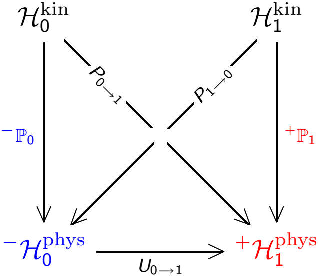


- propagator ansatz for evolution $0 \rightarrow 1$

$$K_{0\rightarrow 1}(x_0, x_1) = M_{0\rightarrow 1}(x_0, x_1)\, e^{iS_1(x_0,x_1)}$$

- e.g. projector $P_{0\rightarrow 1}$

$$^{+}\psi_1^{\rm phys} = P_{0\rightarrow 1}\,\psi_0^{\rm kin} = \int dx_0\, K_{0\rightarrow 1}\,\psi_0^{\rm kin}$$



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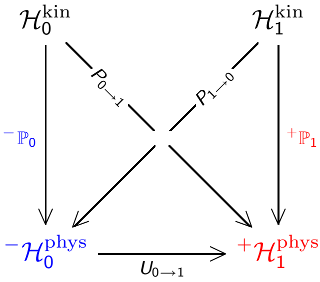

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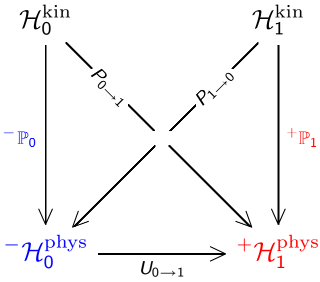

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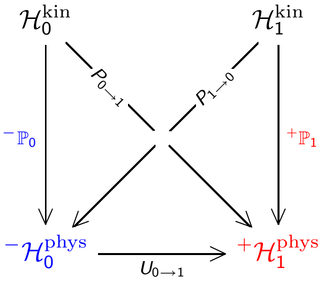

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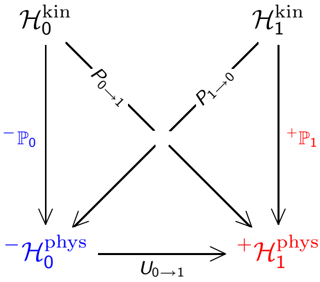

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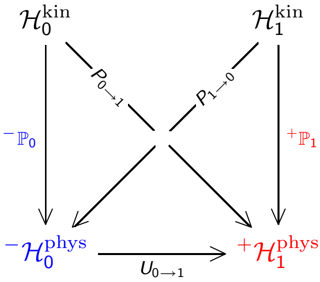

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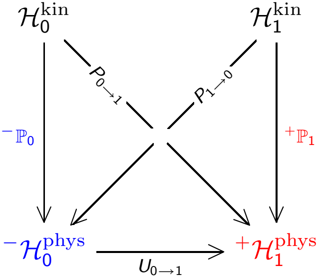

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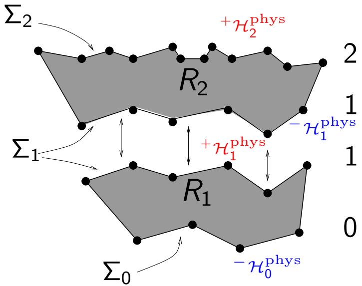
- $U_{0\to1}$ unitary: $\langle {}^{+}\psi_1^{\text{phys}} | {}^{+}\xi_1^{\text{phys}} \rangle_{\text{phys}} = \langle {}^{-}\psi_0^{\text{phys}} | {}^{-}\xi_0^{\text{phys}} \rangle_{\text{phys}}$

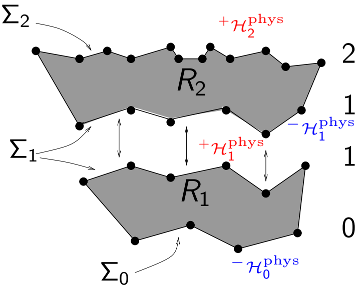


non-trivialities arise when gluing 2 regions

⇒ amounts to concatenation of propagators

$$K_{0\to 2} = \int dx_1 \, K_{1\to 2} \, K_{0\to 1}$$

Composition of global moves [PH '14]



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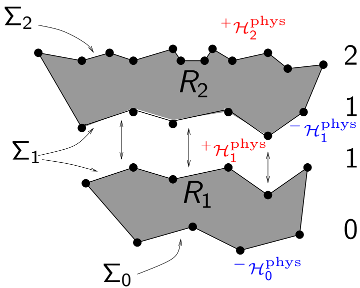

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- recall constraint classification at 1:
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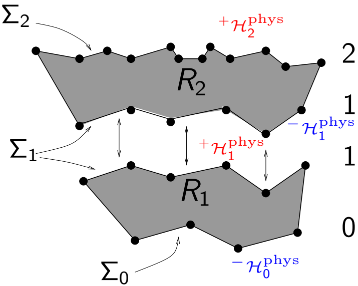

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$\Rightarrow$ amounts to concatenation of propagators

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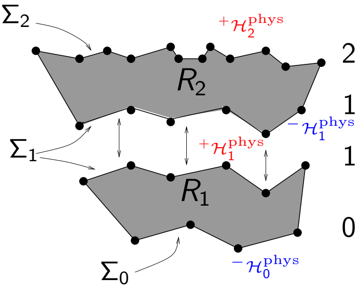
Composition of global moves [PH '14]



non-trivialities arise when gluing 2 regions

- recall constraint classification at 1:
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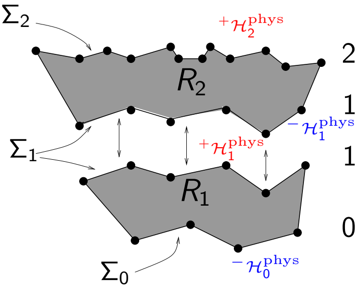
"$(\mathbb{P}_1)^2 \to \infty$" (integration over non-compact gauge orbit)



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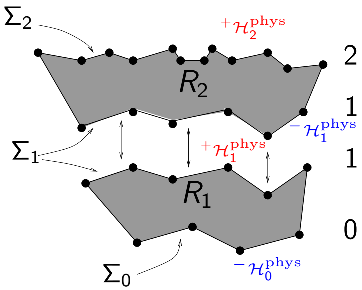
"$(\mathbb{P}_1)^2 \to \infty$" (integration over non-compact gauge orbit)
$\Rightarrow$ regularize by dropping one instance of $\mathbb{P}_1$

Composition of global moves [PH '14]



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- new effective constraints ${}^-\widehat{\tilde{C}}^0$, ${}^+\widehat{\tilde{C}}^2$ also arise in QT

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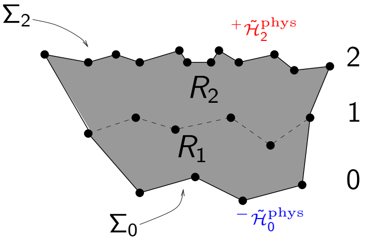

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$\Rightarrow$ non-unitary projections physical Hilbert spaces ${}^+\tilde{\mathcal{H}}_2^{\rm phys} := {}^+\tilde{\mathbb{P}}_2 \, ({}^+\mathcal{H}_2^{\rm phys})$ and ${}^-\tilde{\mathcal{H}}_0^{\rm phys} := {}^-\tilde{\mathbb{P}}_0({}^-\mathcal{H}_0^{\rm phys})$

$\Rightarrow$ non-trivial dynamical coarse graining of discretization

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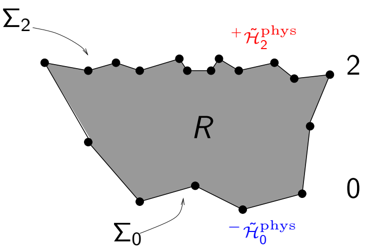

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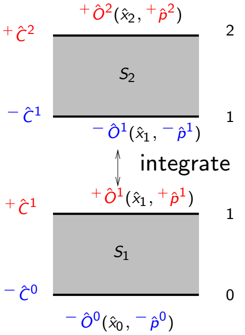
$\Rightarrow$ non-trivial dynamical coarse graining of discretization

$\Rightarrow$ physical Hilbert spaces associated to (boundary) of region (rather than time step) as in 'general boundary formulation' [Oeckl '03, '08]

Physical dofs on varying discretizations [Dittrich, PH '13; PH '14]

- propagating dofs must commute with constraints, to be well-defined on ${}^{-}\mathcal{H}_1^{\rm phys}/{}^{+}\mathcal{H}_1^{\rm phys}$

1. pre–observables ${}^{-}\hat{O}^1$: $[{}^{-}\hat{C}^1_i, {}^{-}\hat{O}^1] = 0$
2. post–observables ${}^{+}\hat{O}^1$: $[{}^{+}\hat{C}^1_i, {}^{+}\hat{O}^1] = 0$



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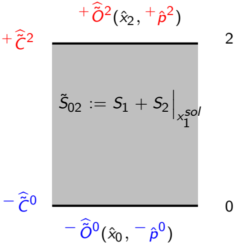


- integrating out 1: ${}^{-}\widehat{\tilde{O}}^0/{}^{+}\widehat{\tilde{O}}^2$ must commute with new ${}^{-}\widehat{\tilde{C}}^0, {}^{+}\widehat{\tilde{C}}^2$

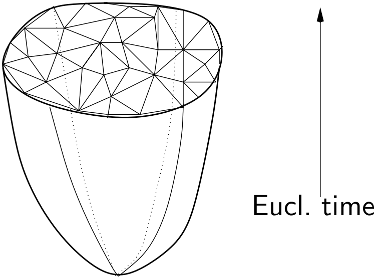
$\Rightarrow$ 'too finely grained' dofs do not commute with new constraints

$\Rightarrow$ dynamical coarse graining projects out dofs irreversibly

- idea: newly added modes ‘Euclideanized’, born in vacuum

- (almost) trivial toy model:
  ‘nothing’ → scalar field on single vertex

$$S_1 = \frac{1}{2}\phi_1^2 \qquad \rightarrow \qquad S_1^{\mathrm{eucl}} = \frac{1}{2} i \phi_1^2$$



post–constraint as annihilation operator:

$$i^+ \hat{C}^1_{\mathrm{eucl}} = \hat{a} = \hat{\phi}_1 + i\hat{p}^1$$

⇒ post–physical state is unique (Gaussian) vacuum state

$${}^+\psi_1^{\mathrm{phys}} \sim e^{-\frac{1}{2}\phi_1^2}$$

- more complicated for large lattice

Discretization changing dynamics in QG [Dittrich, Steinhaus '13, PH '14]

so far:

states evolve in background time (e.g. lattice field theory)

in QG:

physical states 'timeless', do not evolve in external time

$\Rightarrow$ time evolution relational

$\Rightarrow$ discrete state evolution not 'time evolution'

$\Rightarrow$ discretization changing dynamics = coarse graining/refining

3D

- diffeo sym. perserved
- 'evolution' as change of representation

4D

- sym. broken
- non-unitary coarse graining
- refining non-hyperbolic

$\Rightarrow$ need 'dynamical cylindrical consistency' as in Dittrich '12, Dittrich, Steinhaus '13

Summary and Outlook

- systematic classical and quantum formalism for discretization changing dynamics available
- constraints, observables, Hilbert spaces,... region dependent
- non-trivial coarse graining $\Rightarrow$ non-unitary projections of physical Hilbert spaces and observables
- analogously with Pachner move dynamics
- goal: better understand
  1. discretization changing dynamics in QG
  2. 'vacuogenesis'

further reading: PH arXiv: 1401.6062, 1401.7731, B. Dittrich, PH arXiv:1303.4294, 1108.1974