# Elements of quantum theory from limited information and complementarity

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# What makes quantum theory special?

- which physical statements characterize QT within landscape of probabilistic theories?

$\Rightarrow$ wave of $QT$ reconstructions within 'generalized probabilistic theories' framework

[Hardy, Masanes, Müller, Brukner, Dakic, D'Ariano, Chiribella, Perinotti......]

- complementary to this: understand quantum theory as framework for information inference [Rovelli, Zeilinger, Brukner, Fuchs, Spekkens,......]

$\Rightarrow$ Can one characterize/derive QT with primacy on information inference?

Why useful?

1. gain novel perspective on QT within inference theory space
2. why or why not QT in its present form a fundamental theory

# Operational setup

Observer $O$ interrogating system $S$ with *binary* questions $Q_i$, $i = 1, \ldots$

- each $Q_i$ non-trivial 1-bit question (info measure later)
- $O$ has tested identical $S$ sufficiently often to have gained info about set of possible "states"
- Bayesian viewpoint: for specific $S$, $O$ assigns probabilities $p_i$ to $Q_i$ accord. to his info about
  - particular $S$, and
  - about possible set of "states"
- $p_i$ encode all $O$ can say about $S$

$\Rightarrow$ state of $S$ (rel. to $O$): collection of $p_i$

# Question types

- require symmetry under relabeling 'yes' $\leftrightarrow$ 'no' for any $Q_i$
- $\exists$ special state of 'no information' $p_i = \frac{1}{2}\ \forall\, i \Rightarrow$ call *totally mixed state*
- $Q_i, Q_j$ are:

independent: if, relative to totally mixed state of $S$, answer to only $Q_i$ gives $O$ no information about answer to $Q_j$ (and vice versa) $\Rightarrow p(Q_i, Q_j) = p_i \cdot p_j$ factorizes

compatible: if $O$ may know answers to both simultaneously $\Rightarrow p_i, p_j$ can be simultaneously $0, 1$

complementary: if knowledge of $Q_i$ disallows $O$ to know $Q_j$ at the same time (and vice versa) $\Rightarrow p_i = 0, 1$, then $p_j = 1/2$

# Basic postulates (for $N$ qubit systems)

for now, forget about QT, postulate:

LI (limited information): "$O$ can acquire maximally $N \in \mathbb{N}$ independent bits of information about $S$ at the same time."

$\exists\, Q_i,\ i = 1, \ldots, N$ (mutually) independent compatible

C (complementarity): "$O$ can always get up to $N$ new (independent) bits of information about $S$. Whenever $O$ asks a new question he experiences no net loss of information."

$\exists\, Q'_i,\ i = 1, \ldots, N$ independent compatible but $Q_i, Q'_{j=i}$ complementary

(Postulates motivated by Rovelli's Relational Quantum Mechanics and work by Brukner/Zeilinger)

$\Rightarrow$ postulates imply elements of qubit QT

# Number of degrees of freedom

How many independent $Q_i$ for $N$ qubits?

$N = 1$: only individual $Q_i$, $i = 1, \ldots, D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

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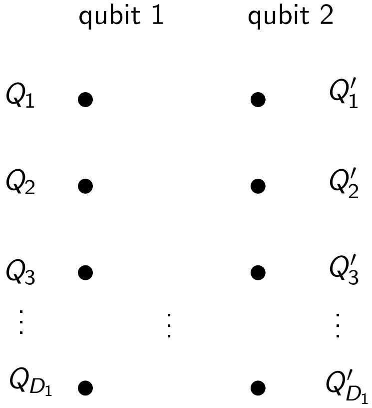
$N = 2$: $2D_1$ individual $Q_i$

vertex: individual question $Q_i, Q'_j$



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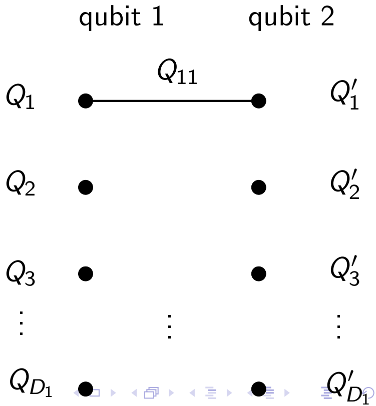
$Q_{ij} := Q_i \leftrightarrow Q'_j$ "Are answers to $Q_i$ and $Q'_j$ the same?"

+ ???

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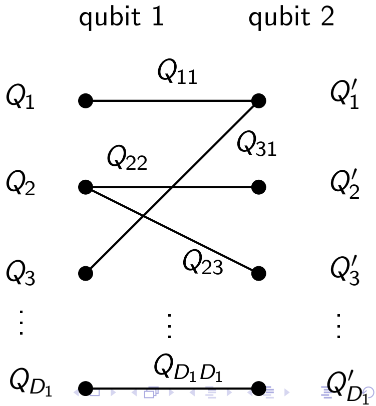

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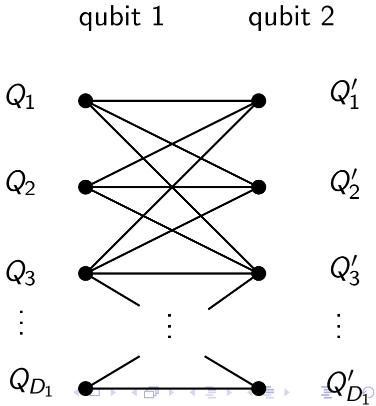

# Compatibility and independence of composite questions

- assume Specker's principle: if $n$ $Q_i$ are pairwise compatible then they are also mutually compatible
- Specker's principle + Complementarity $\Rightarrow \{Q_{ij}\}_{i,j=1}^{D_1}$ pairwise independent

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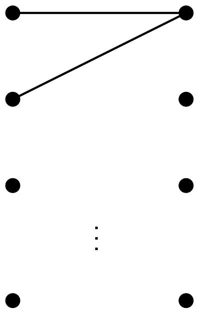

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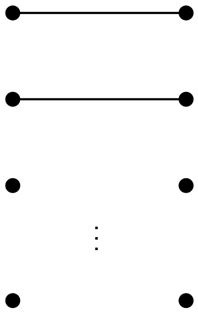
$\Rightarrow$ entanglement: $> 1$ bit in $Q_{ij}$ [Brukner, Zeilinger]



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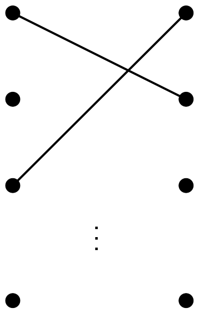

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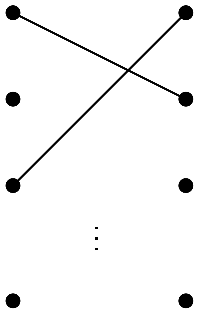
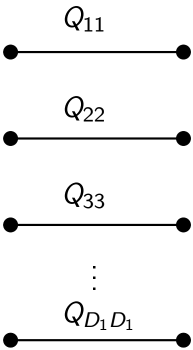
- how many independent $Q$s for $N = 2$?
  $\Rightarrow$ depends on $D_1$



# Why is the Bloch sphere 3-dim.? [see also Müller, Masanes, v. Weizsaecker,....]

Logical argument from $N = 2$ case:

- $Q_{ii}$, $i =, \ldots, D_1$ pairwise independent, compatible
- $O$ can acquire answers to all $D_1$ composites $Q_{ii}$ simultaneously (Specker)



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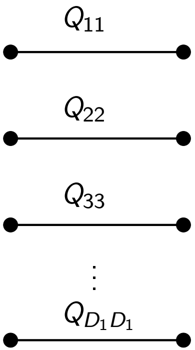

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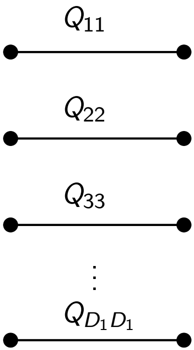

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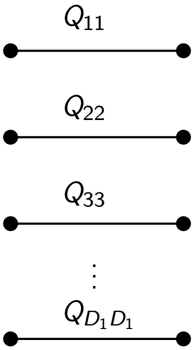
- e.g., truth table for any three $Q_{ii}$ $(a \neq b)$:
  $\Rightarrow Q_{33} = Q_{11} \leftrightarrow Q_{22}$ or $\neg(Q_{11} \leftrightarrow Q_{22})$




| $Q_{11}$ | $Q_{22}$ | $Q_{33}$ |
|---|---|---|
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| 1 | 0 | a |
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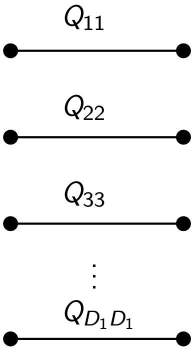


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$2 \leq D_1 \leq 3$

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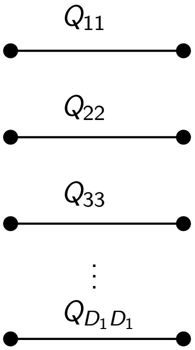


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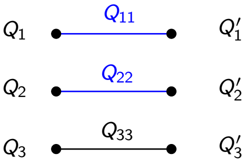
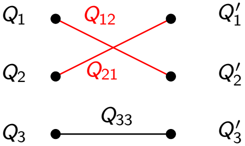
$\Rightarrow$ # DoFs: 15 if $D_1 = 3$; 9 if $D_1 = 2$

# Odd and even correlations

- know: $Q_{33} \overset{?}{=} \neg(Q_{11} \leftrightarrow Q_{22}) \overset{?}{=} \neg(Q_{12} \leftrightarrow Q_{21})$




Hence,

(a) $Q_{11} \leftrightarrow Q_{22} = Q_{12} \leftrightarrow Q_{21}$, or

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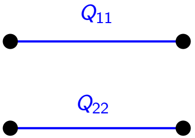


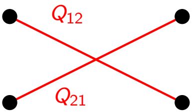

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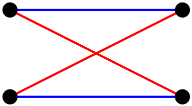
$\Rightarrow$ diagrams can be consistently joined



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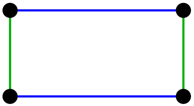

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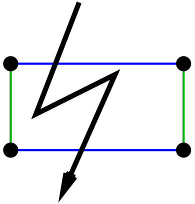

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$\Rightarrow$ illegal complementary info

$\Rightarrow$ would obtain same diagram in 'hidden variable model'

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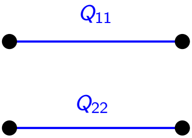


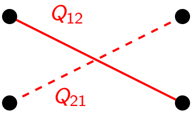

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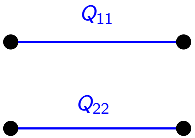
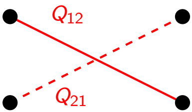


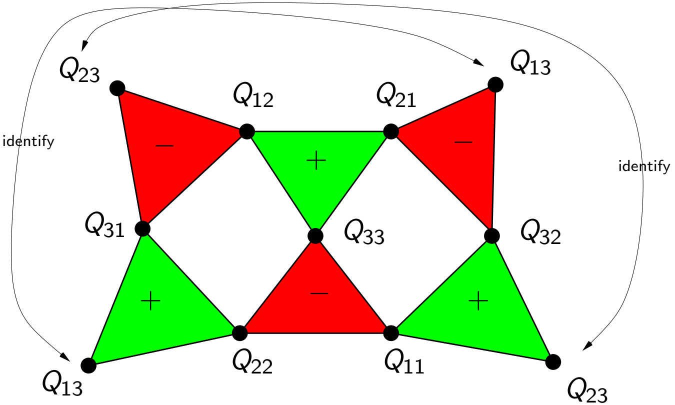

$\Rightarrow$ no illegal info can be extracted

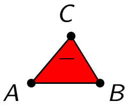
$\Rightarrow$ Complementarity implies (b) $Q_{11} \leftrightarrow Q_{22} = \neg(Q_{12} \leftrightarrow Q_{21})$

# Correlation structure for qubits ($N = 2$ and $D_1 = 3$)

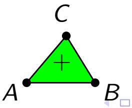
Compatibility structure of $Q$s $\Rightarrow$ correlation structure for 2 qubits in QT

$Q, Q'$ compatible if connected by edge, otherwise complementary




$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...




$\Leftrightarrow$ even correlation
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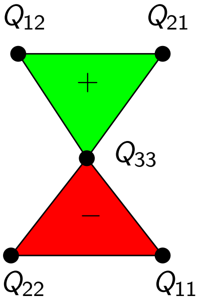
etc...

# Correlation structure for rebits ($N = 2$ and $D_1 = 2$)

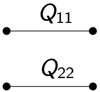
similarly for 2 rebits



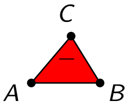

key difference rebits vs. qubits:

$Q_{33} = \neg(Q_{11} \leftrightarrow Q_{22})$

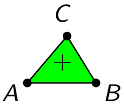
- non-local ($\nexists\ Q_3, Q_3'$)




- complementary to all indiv. Qs
  $\Rightarrow$ determines entanglement




$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...

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# $N = 3$ qubits and monogamy

$(3+1)^3 - 1 = 63$ independent questions (for $D_1 = 3$):

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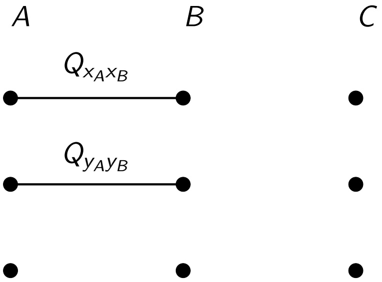

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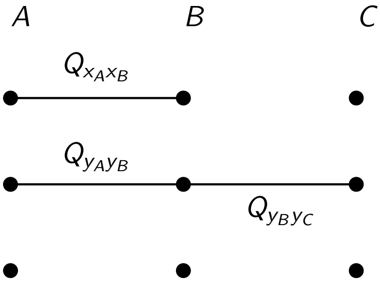

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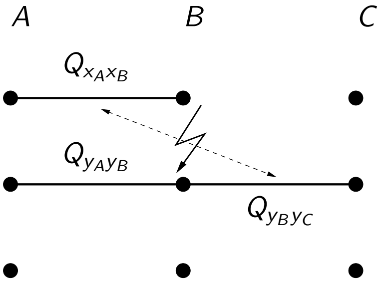

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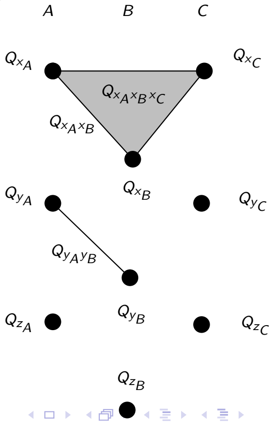

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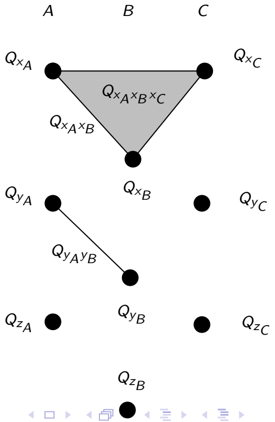
$\Rightarrow$ equivalent to asking $Q_{x_C}, Q_{y_C}$ or $Q_{z_C}$ $\Rightarrow$ only indiv. info about $C$



# Reconstruction attempt of QT

State of $S$ relative to $O$:

$$\vec{P}_{O\to S} = \begin{pmatrix} p_1 \\ \vdots \\ p_{D_N} \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = 1, Q_i \text{ indep.}$$

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- further results: reversible time evolution, Bloch sphere for $N = 1$, results concerning information distribution,....

# Conclusion and outlook

- quantum theory as operational framework for information inference
- quantum state as state of information of $O$ about $S$
- what are physical statements that single out QT from set of inference theories?

$\Rightarrow$ limited information and complementarity imply many structural features of QT

## Outlook:

- complete approach to full reconstruction
- statements characterizing QT $\Rightarrow$ justify why or why not to modify QT for "fundamental theory"?
- in quantum gravity/cosmology 'wave fct. of the universe' as global state of information?
  or rather: no global/absolute state and universe as 'information exchange network' of subsystems