Matter Bounce Scenario in $F(T)$ gravity

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Introduction

- $F(T)$ gravity in flat FLRW geometry
  - Weitzenböck space-time
  - Friedmann equation in $F(T)$ gravity (flat FLRW geometry)
  - Relation with Loop Quantum Cosmology (flat FLRW geometry)

- Matter Bounce Scenario (MBS)
  - MBS as an alternative to inflation
  - The simplest model: dynamics
  - Properties of the simplest model

- Perturbations in $F(T)$ gravity
  - Dynamical equations for perturbations
  - Power spectrum and tensor/scalar ratio in MBS
  - Numerical results

- MBS for different potentials: numeric analysis
  - Matching with a power law or plateau potential
  - Matching with a quintessence potential
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Teleparallelism is based in Weitzenböck space-time.

● Global system of four orthonormal vector fields \( \{ e_i \} \) in the tangent vector bundle.

● Covariant derivative \( \nabla \) that defines absolute parallelism with respect the global basis \( \{ e_i \} \), that is, \( \nabla e_i = 0 \).

Properties of Weitzenböck space-time.

● The connection is metric, i.e., it satisfies \( \nabla g = 0 \).

● Is curvature-free (Riemann tensor vanishes) but has torsion!!!.

● The main invariant is the scalar torsion, namely \( T \).

● For a flat FLRW geometry is given by \( T = -6H^2 \).
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Friedmann eq. in $F(\mathcal{T})$ gravity (flat FLRW geometry)

- **Lagrangian:** $\mathcal{L}_\mathcal{T} = \mathcal{V}(F(\mathcal{T}) + \mathcal{L}_M)$.
  - $\mathcal{V} = a^3$ is the volume.
  - $\mathcal{L}_M$ is the matter Lagrangian density.

- **Hamiltonian:** $\mathcal{H}_\mathcal{T} = \left(2\mathcal{T} \frac{dF(\mathcal{T})}{dT} - F(\mathcal{T}) + \rho\right) \mathcal{V}$.
  - $\rho$ is the energy density.

- **Modified Friedmann equation:** The Hamiltonian constrain $\mathcal{H}_\mathcal{T} = 0$ leads to the equation (a curve in the plane $(H, \rho)$)

  $$\rho = -2 \frac{dF(\mathcal{T})}{dT} \mathcal{T} + F(\mathcal{T}) \equiv G(\mathcal{T}).$$

- **Inverse problem:** Given a curve of the form $\rho = G(\mathcal{T})$ the corresponding $F(\mathcal{T})$ theory is

  $$F(\mathcal{T}) = -\frac{\sqrt{-\mathcal{T}}}{2} \int \frac{G(\mathcal{T})}{\mathcal{T} \sqrt{-\mathcal{T}}} d\mathcal{T}.$$
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Relation with LQC (flat FLRW geometry)

- **Curve (Ellipse):** Friedmann equation in LQC

\[ H^2 = \frac{\rho}{3} \left( 1 - \frac{\rho}{\rho_c} \right), \quad \text{ellipse in the plane } (H, \rho). \]

- **Splitting in two branches:**
  - \( \rho = G_- (\mathcal{T}) \) (branch with \( \dot{H} < 0 \)).
  - \( \rho = G_+ (\mathcal{T}) \) (branch with \( \dot{H} > 0 \)).
  - \( G_\pm (\mathcal{T}) = \frac{\rho_c}{2} \left( 1 \pm \sqrt{1 + \frac{2\mathcal{T}}{\rho_c}} \right). \)

We obtain the model [Amorós et al., PRD 87, [arXiv:1108.0893]]

\[ F_\pm (\mathcal{T}) = \pm \sqrt{-\frac{\mathcal{T} \rho_c}{2}} \arcsin \left( \sqrt{-\frac{2\mathcal{T}}{\rho_c}} \right) + G_\pm (\mathcal{T}). \]
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MBS as an alternative to inflation

The Matter Bounce Scenario: Brief description

It depicts, at early times, a matter dominated Universe in the contracting phase, after the bounce it enters in the expanding one where it matches with the hot Friedmann Universe.

- **Horizon problem solved:** All the parts of the Universe are in causal contact at bouncing time.
- **Flatness problem improved:** The spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding one.
- **Power spectrum:** The power spectrum of cosmological perturbations is scale invariant.
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The simplest model: dynamics

Matter dominated Universe: Dynamics

\[ a(t) = \left( \frac{3}{4} \rho_c t^2 + 1 \right)^{1/3} . \]

Matter scalar field

- Potential of the simplest model: \( V(\bar{\phi}) = 2\rho_c \frac{e^{-\sqrt{3}\bar{\phi}}}{(1+e^{-\sqrt{3}\bar{\phi}})^2} \). 
- Dynamical equation: \( \ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + \frac{\partial V(\bar{\phi})}{\partial \bar{\phi}} = 0 \). 
- Analytic solution:

\[ \bar{\phi}(t) = \frac{2}{\sqrt{3}} \ln \left( \sqrt[4]{\frac{3}{4}} \rho_c t + \sqrt[4]{\frac{3}{4}} \rho_c t^2 + 1 \right) \iff a(t) = \left( \frac{3}{4} \rho_c t^2 + 1 \right)^{1/3} . \]
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The Universe bounces when it reaches black curves defined by $\rho = \rho_c$.

The point $(0, 0)$ is a saddle point, red (resp. green) curves are the invariant curves in the contracting (resp. expanding) phase.

The blue curve corresponds to an orbit.

Before (resp. after) the bounce the blue curve does not cut the red (resp. green) curves.
Phase Portrait of the model $V(\bar{\varphi}) = 2\rho_c \frac{e^{-\sqrt{3}\bar{\varphi}}}{(1+e^{-\sqrt{3}\bar{\varphi}})^2}$

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Perturbations in $F(\mathcal{T})$ gravity

- Dynamical equations for perturbations
- Power spectrum and tensor/scalar ratio in MBS
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- **Dynamical equations**

\[
\zeta''_{S(T)} - c_s^2 \nabla^2 \zeta_{S(T)} + \frac{Z'_{S(T)}}{Z_{S(T)}} \zeta'_{S(T)} = 0,
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[Cai et al., Class. Quant. Grav. 28, [arXiv:1104.4349]] where

- \( c_s^2 = |\Omega| \frac{\arcsin\left(2\sqrt{\frac{3}{\rho_c}}\right)}{2\sqrt{\frac{3}{\rho_c}}} \), with \( \Omega = 1 - \frac{2\rho}{\rho_c} \).
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- **Comparison with Holonomy Corrected LQC**

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Power spectrum and tensor/scalar ratio in MBS

- **Bunch-Davies vacuum:** \( \zeta_T(\eta) = \sqrt{3} \zeta_S(\eta) = \frac{e^{-ik\eta}}{\sqrt{2ka}} \left( 1 - \frac{i}{k\eta} \right) \)
  when \( \eta \to -\infty \)

- **Long wavelength approximation:**
  \[
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F(T) gravity in flat FLRW geometry
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MBS for different potentials: numeric analysis

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Calculations with the analytic solution

- **Power spectrum in \(F(T)\) gravity:** [Haro, JCAP 11, arXiv:1309.0352] \(P_S(k) = \frac{16}{9} \frac{\rho_c}{\rho_{pl}} C^2\),
  \(C = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \ldots = 0.915965...\) Catalan’s constant).

- **Power spectrum in holonomy corrected LQC:** [Wilson-Ewing, JCAP 03, arXiv:1211.6269] \(P_S(k) = \frac{\pi^2}{9} \frac{\rho_c}{\rho_{pl}}\).

- **Tensor/scalar ratio in \(F(T)\) gravity:** \(r = 3 \left( \frac{Si(\pi/2)}{C} \right)^2 \approx 6.7187\),
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  \[ P_S(k) = \frac{\pi^2}{9} \frac{\rho_c}{\rho_{pl}}. \]

- **Tensor/scalar ratio in \( F(T) \) gravity:**
  \[ r = 3 \left( \frac{Si(\pi/2)}{C} \right)^2 \cong 6.7187, \]
  where \( Si(x) \equiv \int_0^x \frac{\sin y}{y} dy \) is the Sine integral function.

- **Tensor/scalar ratio in holonomy corrected LQC:**
  \[ r \cong 0. \]
Tensor/scalar ratio:

\[ r = \frac{1}{3} \left( \frac{\int_{-\infty}^{\infty} \frac{1}{a(t)Z_T(t)} dt}{\int_{-\infty}^{\infty} \frac{1}{a(t)Z_S(t)} dt} \right)^2. \]

Calculations with the analytic solution

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Numerical results

[Haro and Amorós, [arXiv:1403.6396]]

Figure: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\phi}$. First picture for $F(T)$ gravity and second for holonomy corrected LQC.
Minimum value of the power spectrum in $F(T)$ gravity

$\mathcal{P}_S(k) \approx 40 \times 10^{-3} \frac{\rho_c}{\rho_{pl}}$ obtained at $\bar{\varphi} \approx -0.9892$.

Minimum value of the power spectrum in holonomy corrected LQC $\mathcal{P}_S(k) \approx 23 \times 10^{-3} \frac{\rho_c}{\rho_{pl}}$ obtained at $\bar{\varphi} \approx -0.9870$.

Matching with BICEP2 data

- In $F(T)$ gravity $r = 0.20^{+0.07}_{-0.05}$ is realized by solutions bouncing when $\bar{\varphi}$ belongs in $[-1.162, -1.144] \cup [1.144, 1.162]$.
- In holonomy corrected LQC Never.

Matching with Planck’s data

- In $F(T)$ gravity $r \leq 0.11$ is realized by solutions bouncing when $\bar{\varphi}$ belongs in $[-1.205, -1.17] \cup [1.17, 1.205]$.
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MBS for different potentials: numeric analysis

- Matching with a power law
- Matching with a quintessence potential
- Matching with a plateau potential
Matching with a power law potential

Matching with a quadratic potential

Figure: First picture: shape of $\rho_c e^{-\sqrt{3}|\varphi|}$ matched with $\rho_c (\varphi - \varphi_0)^2$. Second picture: phase portrait.
Matching with a quadratic potential

**Figure:** Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\phi}$. In first picture for $F(\mathcal{T})$ gravity, and in the second one for holonomy corrected LQC.
Matching with a quartic potential

Figure: First picture: shape of $\rho_c e^{-\sqrt{3}|\varphi|}$ matched with $\rho_c (\varphi - \varphi_0)^4$. Second picture: phase portrait.
Matching with a quartic potential

Figure: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$, for a potential obtained matching $\rho c e^{-\sqrt{3}|\varphi|}$ with $\rho c (\varphi - \varphi_0)^4$. In first picture for $F(T)$ gravity, and in the second one for holonomy corrected LQC.
Matching with a plateau potential

Figure: Shape and phase space portrait of a potential obtained matching the exponential potential $\rho_c e^{-\sqrt{3} |\varphi|}$ with a plateau potential.
Matching with a plateau potential

Figure: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$, for a potential obtained matching $\rho_c e^{-\sqrt{3}|\varphi|}$ with a plateau potential. In first picture for $F(\mathcal{T})$ gravity, and in the second one for holonomy corrected LQC.
Matching with a quintessence potential

Figure: Shape and phase space portrait of a potential that has matter domination at early times in the contracting phase and quintessence at late times in the expanding phase.
Matching with a quintessence potential

Figure: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\phi}$. In the first picture for $F(T)$ gravity, and in the second one for holonomy corrected LQC.