Friedmann Equation and Emergence of Cosmic Space

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- Pure de Sitter universe satisfies the holographic equipartition

$$N_{\mathtt{sur}} = N_{\mathtt{bulk}}$$

$N_{\mathtt{sur}}$: DOF of boundary, $N_{\mathtt{bulk}}$: DOF of bulk

- Padmanabhan's conjecture [arXiv:1206.4916]:

Assuming our universe is asymptotically de Sitter, the expansion of our universe is being driven towards the holographic equipartition

$$\frac{dV}{dt} = L_p^2 (N_{\mathtt{sur}} - N_{\mathtt{bulk}})$$

V: volume of cosmic space enclosed by apparent horizon
L_p: Planck length

- (n+1)-dimensional FRW universe

The metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right)$$

$k = +1, 0, -1$ correspond to closed, flat, and open universe

This can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_{n-1}^2,\ \ a, b = 0, 1$$

where $\tilde{r} = a(t)r$, $h_{ab} = \texttt{diag}(-1, a^2/1 - kr^2)$, $(x^0, x^1) = (t, r)$

Radius of the apparent horizon:

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}$$

$H \equiv \dot{a}/a$: Hubble parameter

- Flat Einstein case

Volume in the flat case ($k = 0$) enclosed by $\tilde{r}_A$: $V = \Omega_n \tilde{r}_A^n$

Rate of volume change:

$$\frac{dV}{dt} = n\Omega_n \tilde{r}_A^{n-1} \dot{\tilde{r}}_A$$

Friedmann equation in (n+1)-dimensional Einstein gravity:

$$H^2 + \frac{k}{a^2} = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho + np]$$

The Hawking temperature associated with the apparent horizon:

$$T_H = \frac{1}{2\pi \tilde{r}_A}$$

Thus, we get

$$\frac{dV}{dt} = L_p^{n-1}\left(\frac{A}{L_p^{n-1}} + \frac{8\pi\tilde{r}_A V}{n-1}[(n-2)\rho + np]\right)$$

where $A = n\Omega_n \tilde{r}_A^{n-1}$

The bulk Komar energy in (n+1)-dimensional flat spacetime

$$E = \frac{[(n-2)\rho + np]}{(n-2)} V$$

Using the equipartition rule of energy, the bulk DOF is given by

$$N_{\mathtt{bulk}} = \frac{2|E|}{T_H} = -4\pi\tilde{r}_A V \frac{[(n-2)\rho + np]}{n-2}$$

The surface DOF can be identified as

$$N_{\mathrm{sur}} = \alpha \frac{A}{L_p^{n-1}}$$

where $\alpha = (n-1)/2(n-2)$.

Thus, one can write

$$\frac{dV}{dt} = \tilde{L}_p^{n-1}(N_{\mathrm{sur}} - N_{\mathrm{bulk}})$$

with $\tilde{L}_p^{n-1} \equiv L_p^{n-1}/\alpha$.

Note that for $n = 3$, α becomes 1 thus $\tilde{L}_p$ becomes L_p.

• Flat non-Einstein cases

(1) Gauss-Bonnet case

Friedmann equation:

$$H^2 + \frac{k}{a^2} + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2 = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho$$

$$\begin{aligned}\left(\dot{H} - \frac{k}{a^2}\right)\left[1 + 2\tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)\right] &+ \left(H^2 + \frac{k}{a^2}\right)\left[1 + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)\right] \\ &= -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho + np]\end{aligned}$$

where $\tilde{\alpha} = (n-2)(n-3)\alpha$.

The rate of the volume change

$$\frac{dV}{dt} = \frac{L_p^{n-1}}{(1+2\tilde{\alpha}\tilde{r}_A^{-2})}\left(\frac{A(1+\tilde{\alpha}\tilde{r}_A^{-2})}{L_p^{n-1}} + \frac{8\pi\tilde{r}_A V}{n-1}[(n-2)\rho + np]\right)$$

The bulk DOF is given as in the Einstein case

$$N_{\mathtt{bulk}}^{GB} = -4\pi\tilde{r}_A V\frac{[(n-2)\rho + np]}{n-2}$$

The surface DOF is given by

$$N_{\mathtt{sur}}^{GB} = \frac{A(1+\tilde{\alpha}\tilde{r}_A^{-2})}{\tilde{L}_p^{n-1}}$$

Then,

$$\frac{dV}{dt} = \frac{\tilde{L}_p^{n-1}}{(1+2\tilde{\alpha}\tilde{r}_A^{-2})}(N_{\mathtt{sur}}^{GB} - N_{\mathtt{bulk}}^{GB})$$

Cai [2012] introduced the effective volume related to the effective area, which has been used for the entropy of black holes in the Gauss-Bonnet gravity

$$\tilde{A} = A\left(1 + \frac{n-1}{n-3} 2\tilde{\alpha}\tilde{r}_A^{-2}\right)$$

where the effective volume is given by $d\tilde{V}/d\tilde{A} = \tilde{r}_A/(n-1)$
Then,

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1}(N_{\mathtt{sur}}^{GB} - N_{\mathtt{bulk}}^{GB})$$

Mismatch between $\tilde{V}$ in LHS and ordinary V and A in RHS!

If one uses the same bulk and surface DOFs as in the flat Einstein case, then one get the following expression (Yang etal. [2012])

$$\frac{dV}{dt} = L_p^{n-1} \frac{(N_{\mathtt{sur}} - N_{\mathtt{bulk}})/\alpha + \tilde{\alpha} K (N_{\mathtt{sur}}/\alpha)^{1+\frac{2}{1-n}}}{1 + 2\tilde{\alpha} K (N_{\mathtt{sur}}/\alpha)^{\frac{2}{1-n}}}$$

where $K = (n\Omega_n / L_p^{n-1})^{2/(n-1)}$.

(2) Lovelock gravity case

Friedmann equation:

$$\sum_{i=1}^{m} \hat{c}_i \left(H^2 + \frac{k}{a^2} \right)^i = \frac{16\pi L_p^{n-1}}{n(n-1)} \rho$$

$$\left(\dot{H} - \frac{k}{a^2} \right) \sum_{i=1}^{m} i \hat{c}_i \left(H^2 + \frac{k}{a^2} \right)^{i-1} + \sum_{i=1}^{m} \hat{c}_i \left(H^2 + \frac{k}{a^2} \right)^i = -\frac{8\pi L_p^{n-1}}{n(n-1)} [(n-2)\rho + np]$$

where $m = [n/2]$ and $\hat{c}_1 = 1,\ \hat{c}_i = c_i \prod_{j=3}^{2m} (n+1-j)$ for $i > 1$

The rate of volume change

$$\frac{dV}{dt} = \frac{L_p^{n-1}}{\sum_{i=1}^m i\hat{c}_i \tilde{r}_A^{2(1-i)}} \left(\frac{A\sum_{i=1}^m \hat{c}_i \tilde{r}_A^{2(1-i)}}{L_p^{n-1}} + \frac{8\pi \tilde{r}_A V}{(n-1)} [(n-2)\rho + np] \right)$$

The bulk DOF is the same as in the Einstein case

$$N_{\mathrm{bulk}}^L = -4\pi \tilde{r}_A V \frac{[(n-2)\rho + np]}{n-2}$$

The surface DOF is given by

$$N_{\mathrm{sur}}^L = \frac{A}{\tilde{L}_p^{n-1}} \sum_{i=1}^m \hat{c}_i \tilde{r}_A^{2(1-i)}$$

Then,

$$\frac{dV}{dt} = \frac{\tilde{L}_p^{n-1}}{\sum_{i=1}^m i\hat{c}_i \tilde{r}_A^{2(1-i)}} (N_{\mathrm{sur}}^L - N_{\mathrm{bulk}}^L)$$

Introduce the effective area for the entropy of black holes in the Lovelock case

$$\tilde{A} = A \sum_{i=1}^{m} \frac{i(n-1)}{(n-2i+1)} \hat{c}_i \tilde{r}_A^{2(1-i)}$$

where the effective volume is given by $d\tilde{V}/d\tilde{A} = \tilde{r}_A/(n-1)$. Then,

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1}(N_{\mathtt{sur}}^L - N_{\mathtt{bulk}}^L)$$

Again, mismatch between $\tilde{V}$ in LHS and ordinary V and A in RHS!

If one uses the same bulk and surface DOFs as in the flat Einstein case, then one get the following expression (Yang etal. [2012])

$$\frac{dV}{dt} = L_p^{n-1} \frac{(N_{\mathrm{sur}} - N_{\mathrm{bulk}})/\alpha + \sum_{i=2}^{m} \tilde{c}_i K_i (N_{\mathrm{sur}}/\alpha)^{1+\frac{2(i-1)}{1-n}}}{1 + \sum_{i=2}^{m} i\tilde{c}_i K_i (N_{\mathrm{sur}}/\alpha)^{\frac{2(i-1)}{1-n}}}$$

where $K_i = (n\Omega_n/L_p^{n-1})^{2(i-1)/(n-1)}$.

- Non-flat cases

Metric of (3+1)-dimensional non-flat FRW universe

$$V_k = 4\pi a^3 \int_0^{\tilde{r}_A/a} \frac{r^2}{\sqrt{1-kr^2}} dr$$

where $k = \pm 1$, and V_k becomes $4\pi \tilde{r}_A^3/3$ as $k \to 0$.

With the aerial volume for the flat case, $V = \Omega_n \tilde{r}_A^n$, the rate of volume change is given by (Sheykhi [2013])

$$\frac{dV}{dt} = \tilde{L}_p^{n-1} H \tilde{r}_A (N_{\mathtt{sur}} - N_{\mathtt{bulk}})$$

where $V = \Omega_n \tilde{r}_A^n$.

For the Gauss-Bonnet and Lovelock cases, using the similar effective volumes as in the flat cases, similar results were obtained. Mismatches between volumes of LHS and RHS.

With the proper volume, the rate of volume change is given by

$$\frac{dV_k}{dt} = L_p^2\left[\frac{A}{L_p^2}H\tilde{r}_A\frac{V_k}{\bar{V}_k} + \frac{\bar{V}_k}{V_k}4\pi\tilde{r}_A(\rho+3p)V_k\right]$$

where $\bar{V}_k = 4\pi\tilde{r}_A^3/3$ and $A = 4\pi\tilde{r}_A^2$.

The Komar energy in the non-flat case

$$E_k = (\rho+3p)V_k$$

The bulk DOF

$$N_{\mathrm{bulk}} = \frac{2|E_k|}{T_H} = -4\pi\tilde{r}_A(\rho+3p)V_k$$

The surface DOF

$$N_{\mathrm{sur}} = A/L_p^2$$

The rate of volume change with the proper volume

$$\frac{dV_k}{dt} = L_p^2 \left(H\tilde{r}_A \frac{V_k}{\bar{V}_k} N_{\mathtt{sur}} - \frac{\bar{V}_k}{V_k} N_{\mathtt{bulk}} \right) \equiv L_p^2 \Delta \mathcal{N}$$

Note that

$$\Delta N \equiv N_{\mathtt{sur}} - N_{\mathtt{bulk}} = \frac{4\pi \tilde{r}_A^2}{L_p^2} \frac{V_k}{\bar{V}_k} \left[\left(\frac{\dot{\tilde{r}}_A}{H\tilde{r}_A} - 1 + \frac{\bar{V}_k}{V_k} \right) \right]$$

Eune and Kim [2013] rewrote the rate of volumve change as

$$\frac{dV_k}{dt} = L_p^2 f_k(t) \Delta N$$

where

$$f_k(t) \equiv \frac{\Delta \mathcal{N}}{\Delta N} = \frac{L_p^2}{4\pi \tilde{r}_A^2} \frac{\bar{V}_k}{V_k} \frac{\left(H\tilde{r}_A \frac{V_k}{\bar{V}_k} N_{\mathtt{sur}} - \frac{\bar{V}_k}{V_k} N_{\mathtt{bulk}} \right)}{\left(\frac{\dot{\tilde{r}}_A}{H\tilde{r}_A} - 1 + \frac{\bar{V}_k}{V_k} \right)}$$

- Conclusion

Padmanabhan's conjecture that the expansion of the universe is being driven towards holographic equipartition is quite attractive.

However, as we saw, only the flat or the spatially flat Einstein case seems to be compatible with this conjecture.

So, this may suggest why our universe is spatially flat, being dictated by the holography of nature.