Damping of gravitational waves in the nonperturbative spinor vacuum

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Gravitational waves (GWs) are probably the most suitable object for studying the deep space. It is usually assumed that GWs propagate in a classical vacuum, i.e., in empty space. But a quantum vacuum possesses the energy associated with the unavoidable quantum fluctuations of various fields when the vacuum expectation value of any quantum field is zero but the expectation value of the square of fluctuations is nonzero.

In this framework, of special interest is to study the question of the propagation of GWs in the case where fluctuations of a quantum spinor field are taken into account. The reason is that the energy-momentum tensor of a spinor field contains the spin connection, which in turn contains first derivatives of tetrad components with respect to the coordinates. As a result, the Einstein equations yield the wave equation for a GW which contains second derivatives of the tetrad components on the lefthand side and their first derivatives on the righthand side.

Such a situation is a reminder of the propagation of electromagnetic waves in a continuous conducting medium. The corresponding wave equation is

$$\Delta \vec{A} - \frac{\epsilon\mu}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{\gamma\mu}{c^2}\frac{\partial \vec{A}}{\partial t},$$

We consider a GW propagating along the x axis. The Einstein equations are

$$\delta G_{\bar{a}\bar{b}} = \varkappa \left\langle Q \left| \widehat{\delta T_{\bar{a}\bar{b}}} \right| Q \right\rangle ,$$

The operator of the energy-momentum tensor of the spinor field is given by

$$\hat{T}_{\bar{a}\bar{b}} = \frac{i}{2}\left[\hat{\bar{\psi}}\gamma_{(\bar{a}}\nabla_{\bar{b})}\hat{\psi} - \nabla_{(\bar{a}}\hat{\bar{\psi}}\gamma_{\bar{b})}\hat{\psi}\right]$$

The energy-momentum tensor has unperturbed and perturbed contributions

$$\hat{T}_{\bar{a}\bar{b}} = \overset{0}{\hat{T}}_{\bar{a}\bar{b}} + \widehat{\delta T}_{\bar{a}\bar{b}}$$

$\overset{0}{\hat{T}}_{\bar{a}\bar{b}}$ is calculated for unperturbed Minkowski spacetime with zero spin connection $\omega_{\bar{a}\bar{b}\mu} = 0$ and consequently

$$\overset{0}{\hat{T}}_{\bar{a}\bar{b}} = \frac{i}{2}\left[\hat{\bar{\psi}}\gamma_{(\bar{a}}\partial_{\bar{b})}\hat{\psi} - \partial_{(\bar{a}}\hat{\bar{\psi}}\gamma_{\bar{b})}\hat{\psi}\right].$$

Perturbed energy-momentum tensor is calculated as

$$\widehat{\delta T}_{\bar{a}\bar{b}} = -\frac{i}{2}\hat{\bar{\psi}}\left[\gamma_{(\bar{a}}\delta\Gamma_{\bar{b})} + \delta\Gamma_{(\bar{a}}\gamma_{\bar{b})}\right]\hat{\psi}$$

where $\delta\Gamma_{\bar{b}}$ is perturbed spinor connection

Nonperturbative quantization a la Heisenberg

Step 1. $G_{\mu\nu} = \varkappa T_{\mu\nu}$

Step 2. $\hat{G}_{\mu\nu} = \varkappa \hat{T}_{\mu\nu}$

Step 3. $\left\langle \hat{G}_{\mu\nu} \right\rangle = \left\langle \varkappa \hat{T}_{\mu\nu} \right\rangle$

Step 4. $\left\langle \hat{g}^{\alpha\beta} \hat{G}_{\mu\nu} \right\rangle = \varkappa \left\langle \hat{g}^{\alpha\beta} \hat{T}_{\mu\nu} \right\rangle$

...

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$\ldots$

- the metric is classical;
- $\hat{\psi}$ is quantum;
- for $\left\langle \hat{\psi}\hat{\psi} \right\rangle$ we assume some ansatz;
- correctness is verified using the Bianchi identities.

In this case we have the following set of equations for the components

$$
\begin{aligned}
h''_{yy} - \ddot{h}_{yy} &= -2\varkappa \left(\left\langle \hat{A}^* \hat{B} \right\rangle + \left\langle \hat{B}^* \hat{A} \right\rangle \right) \dot{h}_{yz}, \\
h''_{yz} - \ddot{h}_{yz} &= 2\varkappa \left(\left\langle \hat{A}^* \hat{B} \right\rangle + \left\langle \hat{B}^* \hat{A} \right\rangle \right) \dot{h}_{yy},
\end{aligned}
$$

To calculate the expectation value of the energy-momentum tensor of the spinor field, we state the following assumptions concerning the spinor field:

- $$\left\langle Q \left| \hat{\psi}_{\mathfrak{a}} \right| Q \right\rangle = 0. \tag{1}$$
- $$\left\langle Q \left| \hat{\psi}^*_{\mathfrak{a}}(x) \hat{\psi}_{\mathfrak{b}}(y) \right| Q \right\rangle = \Upsilon_{\mathfrak{a}\mathfrak{b}}(x, y) \neq 0. \tag{2}$$
- $$|\Upsilon_{\mathfrak{a}\mathfrak{b}}(x, x)| = \mathrm{const}. \tag{3}$$
- As a consequence of Eq. (3) we have
$$\left\langle Q \left| \hat{\psi}^*_{\mathfrak{a}}(x) \partial_{y^\mu} \hat{\psi}_{\mathfrak{b}}(y) \right| Q \right\rangle = 0. \tag{4}$$

We assume the following values of the 2-point Green's functions of the spinor field ψ:

$$\begin{aligned}\Upsilon &= \langle\psi_1^*\psi_2\rangle = \langle\psi_1^*\psi_3\rangle = \langle\psi_4^*\psi_2\rangle = \langle\psi_4^*\psi_3\rangle = \\ &\quad \langle A^* B\rangle = \Upsilon_1 + i\Upsilon_2, \\ \Upsilon^* &= \langle\psi_2^*\psi_1\rangle = \langle\psi_2^*\psi_4\rangle = \langle\psi_3^*\psi_1\rangle = \langle\psi_3^*\psi_4\rangle = \\ &\quad \langle B^* A\rangle = \Upsilon_1 - i\Upsilon_2\end{aligned}$$

with $\left|\Upsilon_{1,2}\right| = const$.

We are looking for the x-plane wave solution in the form

$$
\begin{aligned}
h_{yy} = -h_{zz} &= A_1 e^{-i(\omega t - kx)}, \\
h_{yz} = h_{zy} &= A_2 e^{-i(\omega t - kx)},
\end{aligned}
$$

Ansatz for the spinor field:

$$
\hat{\psi} = e^{-i(\omega t - kx)} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{B} \\ \hat{A} \end{pmatrix},
$$

GW equations are

$$
\begin{aligned}
A_1\left(k^2-\omega^2\right) &= -4i\varkappa A_2\Upsilon_1\omega, \\
A_2\left(k^2-\omega^2\right) &= 4i\varkappa A_1\Upsilon_1\omega.
\end{aligned}
$$

From them one can immediately read out

$$A_2 = \pm iA_1 = A_1 e^{\pm i\frac{\pi}{2}}.$$

This means that the phase difference between yy, zz and yz components of the GW is $\pm\pi/2$.

$$k^2 = \omega^2 \pm 4\varkappa\Upsilon_1\omega$$

Now check the Bianchi identity

$$\left\langle \hat{T}_{\bar{a}}{}^{\mu} \right\rangle_{;\mu} = 0$$

Conclusions

- There exists the fixed phase difference $\pm\pi/2$ between $h_{yy,zz}$ and h_{yz}.
- $v_{GW} \neq c$.
- $h_{yy,zz}$ and h_{yz} do exist together only.
- Damping of the GW may occur for some frequencies ω.
- For given ω, there exist two GWs with different k.