Coherent State Operators in Cosmology and Gravity

Andrea Dapor

Faculty of Physics, University of Warsaw

Marseille, 18 July 2014

E Alesci, AD, J Lewandowski, I Makinen - in preparation
coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions
coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions
Many definitions of a family of coherent states (CS). But the bare minimum is:

- peakedness
- resolution of identity
Many definitions of a family of coherent states (CS). But the bare minimum is:

- peakedness
- resolution of identity

**Peakedness:** Each CS in the family is labelled by a point \((q, p) \in \Gamma\). It is “peaked” on it, i.e.

\[ \hat{Q} |q, p\rangle \approx q |q, p\rangle, \quad \hat{P} |q, p\rangle \approx p |q, p\rangle \]
Many definitions of a family of *coherent states* (CS). But the bare minimum is:

- peakedness
- resolution of identity

**Peakedness:** Each CS in the family is labelled by a point \((q, p) \in \Gamma\). It is “peaked” on it, i.e.

\[
\hat{Q} |q, p\rangle \approx q |q, p\rangle, \quad \hat{P} |q, p\rangle \approx p |q, p\rangle
\]

In general, CS’s in \([|q, p\rangle]\) are not orthogonal, \(\langle q, p|q', p'\rangle \neq \delta_{qq'}\delta_{pp'}\). E.g., for Gaussian CS’s

\[
\langle x|q, p\rangle = N e^{-\frac{(x-(q-ip))^2}{2t}}
\]

it is

\[
|\langle q, p|q', p'\rangle| \sim \exp\left(-\frac{1}{4t} \left[ (q - q')^2 + (p - p')^2 \right]\right) \xrightarrow{t \to 0} \delta_{qq'}\delta_{pp'}
\]
coherent state “quantization”

**Resolution of Identity**: there exists a measure $d\mu(q, p)$ on $\Gamma$ such that

$$\int d\mu(q, p) |q, p\rangle\langle q, p| = \hat{I}$$

This ensures that the family $\{|q, p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.
Resolution of Identity: there exists a measure $d\mu(q, p)$ on $\Gamma$ such that

$$\int d\mu(q, p) |q, p\rangle\langle q, p| = \hat{I}$$

This ensures that the family $\{|q, p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.

We want to use CS's to build operators, which we call coherent state operators.
Resolution of Identity: there exists a measure $d\mu(q,p)$ on $\Gamma$ such that

$$\int d\mu(q,p) \langle q,p \rangle \langle q,p \rangle = \hat{I}$$

This ensures that the family $\{|q,p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.

We want to use CS's to build operators, which we call coherent state operators.

$\Rightarrow$ Extend the resolution of identity to any function on $\Gamma$:

$$1 \rightarrow \hat{A}_1 := \int d\mu(q,p) 1 |q,p\rangle \langle q,p|$$
Resolution of Identity: there exists a measure $d\mu(q, p)$ on $\Gamma$ such that

$$\int d\mu(q, p) |q, p\rangle\langle q, p| = \hat{I}$$

This ensures that the family $\{|q, p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.

We want to use CS’s to build operators, which we call coherent state operators.

⇒ Extend the resolution of identity to any function on $\Gamma$:

$$F \rightarrow \hat{A}_F := \int d\mu(q, p) F(q, p) |q, p\rangle\langle q, p|$$
Resolution of Identity: there exists a measure $d\mu(q, p)$ on $\Gamma$ such that

$$\int d\mu(q, p) |q, p\rangle\langle q, p| = \hat{I}$$

This ensures that the family $\{|q, p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.

We want to use CS’s to build operators, which we call coherent state operators.

⇒ Extend the resolution of identity to any function on $\Gamma$:

$$F \rightarrow \hat{A}_F := \int d\mu(q, p) F(q, p) |q, p\rangle\langle q, p|$$

Inbuilt semiclassical limit:

$$\langle q, p|\hat{A}_F|q, p\rangle = \int d\mu(q', p') F(q', p') |\langle q, p|q', p'\rangle|^2 = F(q, p) + O(t)$$
canonical vs coherent

Some examples of canonical pairs that we know and love:
Some examples of canonical pairs that we know and love:

\[ x, -i\hbar \frac{d}{dx} \]

\[ \hat{a}, \hat{a}^{\dagger} \]
For canonical variables and some special symmetries, the Poisson algebra carries over to the quantum level:

\[
[\hat{Q}, \hat{P}] = i\hbar \hat{I} \quad \Rightarrow \quad [\hat{J}_1, \hat{J}_2] = i\hbar \hat{J}_3
\]

\[
[\text{Daisy}, \text{Donald}] = i\hbar \quad \Rightarrow \quad [\text{Scrooge}, \text{Donald}] = i\hbar
\]
For canonical variables and some special symmetries, the Poisson algebra carries over to the quantum level:

\[
[\hat{Q}, \hat{P}] = i\hbar \hat{I} \quad \Rightarrow \quad [\hat{J}_1, \hat{J}_2] = i\hbar \hat{J}_3
\]

But this is not true in general:

\[
[\hat{F}, \hat{G}] \neq i\hbar \{\hat{F}, \hat{G}\}
\]
For canonical variables and some special symmetries, the Poisson algebra carries over to the quantum level:

\[
[\hat{Q}, \hat{P}] = i\hbar \hat{I} \quad \Rightarrow \quad [\hat{J}_1, \hat{J}_2] = i\hbar \hat{J}_3
\]

But this is not true in general:

\[
[\hat{F}, \hat{G}] \neq i\hbar \{\hat{F}, \hat{G}\}
\]

So we should not be too sad if CS operators do not respect any canonical relation:

\[
[\hat{A}_F, \hat{A}_G] \neq i\hbar \hat{A}_{\{F,G\}}
\]
coherent state “quantization”

example 1: quantum cosmology

dpexample 2: quantum gravity

conclusions
Classical Theory

Consider homogeneous isotropic cosmology with perfect fluid ($p = w \rho$): with a proper choice of variables (B F Schutz, 1970, 1971)

- fluid ($T, p_T$) $\in \mathbb{R} \times \mathbb{R}^+$
- gravitation ($q, p_q$) $\in \mathbb{R}^+ \times \mathbb{R}$

the Hamiltonian constraint in comoving coordinates $H = -p_T^2 - k \beta (w) q_\mu (w) + p_T$.

$\Rightarrow$ Physical Hamiltonian on gravitational phase space: $h (q, p) = p_T^2 + k \beta (w) q_\mu (w)$.
Classical Theory

Consider homogeneous isotropic cosmology with perfect fluid \((p = w \rho)\): with a proper choice of variables [B F Schutz (1970, 1971)]

- fluid \((T, p_T) \in \mathbb{R} \times \mathbb{R}_+\)
- gravitation \((q, p) \in \mathbb{R}_+ \times \mathbb{R}\)

the Hamiltonian constraint in comoving coordinates

\[
H = -p^2 - k \beta(w) q^{\mu(w)} + p_T
\]
Classical Theory

Consider homogeneous isotropic cosmology with perfect fluid \((p = w\rho)\): with a proper choice of variables \([\text{B F Schutz (1970, 1971)}]\)

- fluid \((T, p_T) \in \mathbb{R} \times \mathbb{R}_+\)
- gravitation \((q, p) \in \mathbb{R}_+ \times \mathbb{R}\)

the Hamiltonian constraint in comoving coordinates

\[
H = -p^2 - k \beta(w) q^{\mu(w)} + p_T
\]

\(\Rightarrow\) Physical Hamiltonian on gravitational phase space:

\[
h(q, p) = p^2 + k \beta(w) q^{\mu(w)}
\]
Quantum Theory (1)
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

\[ \mathcal{H} = L^2(\mathbb{R}_+, dx) \]
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

$$\mathcal{H} = L^2(\mathbb{R}_+, dx)$$

To define the family of CS's, \{\ket{q, p}\}:
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

$$\mathcal{H} = L^2(\mathbb{R}_+, dx)$$

To define the family of CS’s, $\{|q, p\rangle\}$:

- choose a fiducial vector $|\psi_0\rangle \in \mathcal{H}$
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

\[ \mathcal{H} = L^2(\mathbb{R}_+, dx) \]

To define the family of CS's, \( \{|q, p\rangle\} \):

- choose a fiducial vector \( |\psi_0\rangle \in \mathcal{H} \)
- choose a group \( G \) parametrized by \( \Gamma \), and a unitary irrep \( U \) of \( G \) on \( \mathcal{H} \)
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

\[ \mathcal{H} = L^2(\mathbb{R}_+, dx) \]

To define the family of CS’s, \{\ket{q, p}\}:

- choose a fiducial vector \ket{\psi_0} \in \mathcal{H}
- choose a group \(G\) parametrized by \(\Gamma\), and a unitary irrep \(U\) of \(G\) on \(\mathcal{H}\)

Then

\[ \ket{q, p} := U(q, p) \ket{\psi_0} \]
Quantum Theory (1)

Hilbert space of square-integrable functions on the configuration space:

$$\mathcal{H} = L^2(\mathbb{R}_+, dx)$$

To define the family of CS's, $$\{|q, p\rangle\}$$:

- choose a fiducial vector $$|\psi_0\rangle \in \mathcal{H}$$
- choose a group $$G$$ parametrized by $$\Gamma$$, and a unitary irrep $$U$$ of $$G$$ on $$\mathcal{H}$$

Then

$$|q, p\rangle := U(q, p) |\psi_0\rangle$$

In our case, the structure of $$\Gamma = \mathbb{R}_+ \times \mathbb{R}$$ suggests the use of affine group:

$$\langle x|q, p\rangle = (U(q, p) \psi_0)(x) = \frac{e^{ipx}}{\sqrt{q}} \psi_0(x/q)$$
Quantum Theory (2)
Quantum Theory (2)

By Schur's lemma wrt the affine group,

$$\int dq dp \, |q, p\rangle \langle q, p| = c \hat{1}$$
Quantum Theory (2)

By Schur’s lemma wrt the affine group,

$$\int dq \, dp \, |q, p \rangle \langle q, p| = c \hat{I}$$

$$\Rightarrow$$ The family of CS \{|q, p\rangle\} resolves the identity with measure $d\mu(q, p) = dq \, dp / c$.

We can construct CS operators! In particular,

$$\hat{A} p^2 = \hat{P}^2 + K(\psi_0) \hat{Q} - 2,$$

where $K(\psi_0) = \int c \, y \, |\psi'_0(y)|^2 \, dy$.

Only the numerical value of $K$ depends on the choice of fiducial state $\psi_0$.

$$\Rightarrow$$ CS quantization of the kinetic term produces a repulsive potential!

CS Hamiltonian $h = p^2 + k_{\beta}(w) q_{\mu}(w)$.
Quantum Theory (2)

By Schur’s lemma wrt the affine group,

\[ \int dq \, dp \, |q, p\rangle \langle q, p| = c \hat{I} \]

\[ \Rightarrow \] The family of CS \{ |q, p\rangle \} resolves the identity with measure \( d\mu(q, p) = dq \, dp / c \).

We can construct CS operators! In particular,

\[ \hat{A}_p^2 = \hat{P}^2 + K(\psi_0) \hat{Q}^{-2} \]

\[ K(\psi_0) := \int_0^\infty dy \frac{y}{c} |\psi_0'(y)|^2 \]

Only the numerical value of \( K \) depends on the choice of fiducial state \( \psi_0 \).
Quantum Theory (2)

By Schur’s lemma wrt the affine group,

\[ \int dq \, dp \, |q, p\rangle \langle q, p| = c \hat{I} \]

⇒ The family of CS \{ |q, p\rangle \} resolves the identity with measure \( d\mu(q, p) = dq \, dp / c \).

We can construct CS operators! In particular,

\[ \hat{A}_p^2 = \hat{P}^2 + K(\psi_0) \hat{Q}^{-2}, \quad K(\psi_0) := \int_0^\infty \frac{dy}{c} \, y |\psi'_0(y)|^2 \]

Only the numerical value of \( K \) depends on the choice of fiducial state \( \psi_0 \).

⇒ CS quantization of the kinetic term produces a repulsive potential!
Quantum Theory (2)

By Schur’s lemma wrt the affine group,

\[
\int dq \, dp \, |q, p\rangle \langle q, p| = c \hat{I}
\]

⇒ The family of CS \{ |q, p\rangle \} resolves the identity with measure

\[d\mu(q, p) = dq \, dp / c.\]

We can construct CS operators! In particular,

\[
\hat{A}_p^2 = \hat{P}^2 + K(\psi_0) \hat{Q}^{-2}, \quad K(\psi_0) := \int_0^\infty \frac{dy}{c} \, y |\psi'_0(y)|^2
\]

Only the numerical value of \(K\) depends on the choice of fiducial state \(\psi_0\).

⇒ CS quantization of the kinetic term produces a repulsive potential!

CS Hamiltonian

\[
h = p^2 + k \beta(w) q^{\mu(w)}
\]
Quantum Theory (2)

By Schur's lemma wrt the affine group,

$$\int dq dp \langle q, p \rangle |q, p\rangle = c \hat{I}$$

⇒ The family of CS \{\langle q, p \rangle\} resolves the identity with measure $d\mu(q, p) = dq dp / c$.

We can construct CS operators! In particular,

$$\hat{A}_{p^2} = \hat{P}^2 + K(\psi_0) \hat{Q}^{-2}, \quad K(\psi_0) := \int_0^\infty \frac{dy}{c} y |\psi'_0(y)|^2$$

Only the numerical value of $K$ depends on the choice of fiducial state $\psi_0$.

⇒ CS quantization of the kinetic term produces a repulsive potential!

CS Hamiltonian

$$\hat{A}_h = \hat{P}^2 + K(\psi_0) \hat{Q}^{-2} + k \bar{\beta}(w) \hat{Q}^{\mu(w)}$$
Results

For a specific choice of $\psi_0$, the quantum dynamics can be solved analytically.

Semiclassical (effective) dynamics:

$$h_{\text{eff}}(q, p) := \langle q, p | \hat{A}_h | q, p \rangle = p^2 + k A q^2 + B q^2$$

$\Rightarrow$ integral curves of $h_{\text{eff}}$ consistently avoid the $q = 0$ region.
Results

For a specific choice of $\psi_0$, the quantum dynamics can be solved analytically.
Results

For a specific choice of $\psi_0$, the quantum dynamics can be solved analytically.

Semiclassical (effective) dynamics:

$$h_{\text{eff}}(q, p) := \langle q, p | \hat{A}_h | q, p \rangle = p^2 + k A q^2 + B \frac{1}{q^2}$$
Results

For a specific choice of $\psi_0$, the quantum dynamics can be solved analytically.

Semiclassical (effective) dynamics:

$$h_{\text{eff}}(q, p) := \langle q, p|\hat{A}_h|q, p \rangle = p^2 + k A q^2 + B \frac{1}{q^2}$$

$\Rightarrow$ integral curves of $h_{\text{eff}}$ consistently avoid the $q = 0$ region.

(Integral curves of $h$ and $h_{\text{eff}}$ respectively, for $k = 1$.)
coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions
Strucutre of LQG

M = Σ \times R. Hamiltonian formalism: g_{\mu\nu} \rightarrow (q_{ab}, p_{ab}), canonical ADM variables.

Symplectic transformation (to extended Γ) ⇒ canonical AB variables, (A_i^a(x), E_a^i(x)).

- A_i^a(x) is smeared on 1-dim submanifolds l \subset Σ called links: h_l(A) \in SU(2).
- E_a^i(x) is smeared on 2-dim submanifolds S \subset Σ called surfaces: P_S(E) \in su(2) = R^3.

⇒ geometry is "polymerized", and lives on graphs, γ. Hilbert space (of cylindrical functions): H = \bigoplus_γ H_γ, H_γ = L^2(SU(2)_{\mathcal{L}}, d\mathcal{L}_{\mu}H).

Basis of spin-networks for H_γ: ψ_{\vec{j}, \vec{m}, \vec{n}}(\vec{g}) \equiv ⟨\vec{g}|\vec{j}, \vec{m}, \vec{n⟩ = L^\prod_{l=1}^{|L|} \sqrt{d_{j_l}} D(j_l)_{m_l n_l}(g_l)
Strucutre of LQG

\[ M = \Sigma \times \mathbb{R}. \] Hamiltonian formalism: \( g_{\mu\nu} \rightarrow (q_{ab}, p^{ab}) \), canonical ADM variables.
Structure of LQG

\[ M = \Sigma \times \mathbb{R} \]. Hamiltonian formalism: \( g_{\mu\nu} \rightarrow (q_{ab}, p^{ab}) \), canonical ADM variables.

Symplectic transformation (to extended \( \Gamma \)) \( \Rightarrow \) canonical AB variables, \( (A^i_a(x), E^a_i(x)) \).
CS operators in LQG

Structure of LQG

\[ M = \Sigma \times \mathbb{R}. \] Hamiltonian formalism: \( g_{\mu\nu} \rightarrow (q_{ab}, p^{ab}) \), canonical ADM variables.

Symplectic transformation (to extended \( \Gamma \)) \( \Rightarrow \) canonical AB variables, \((A^i_a(x), E^a_i(x))\).

- \( A^i_a(x) \) is smeared on 1-dim submanifolds \( I \subset \Sigma \) called links: \( h_i(A) \in SU(2) \)
- \( E^a_i(x) \) is smeared on 2-dim submanifolds \( S \subset \Sigma \) called surfaces: \( P_S(E) \in \mathfrak{su}_2 = \mathbb{R}^3 \)
CS operators in LQG

Structure of LQG

\( M = \Sigma \times \mathbb{R} \). Hamiltonian formalism: \( g_{\mu\nu} \rightarrow (q_{ab}, p^{ab}) \), canonical ADM variables.

Symplectic transformation (to extended \( \Gamma \)) \( \Rightarrow \) canonical AB variables, \((A^i_a(x), E^a_i(x))\).

- \( A^i_a(x) \) is smeared on 1-dim submanifolds \( l \subset \Sigma \) called links: \( h_l(A) \in SU(2) \)
- \( E^a_i(x) \) is smeared on 2-dim submanifolds \( S \subset \Sigma \) called surfaces: \( P_S(E) \in su_2 = \mathbb{R}^3 \)

\( \Rightarrow \) geometry is “polymerized”, and lives on graphs, \( \gamma \). Hilbert space (of cylindrical functions):

\[ \mathcal{H} = \bigoplus_{\gamma} \mathcal{H}_\gamma, \quad \mathcal{H}_\gamma = L^2(SU(2)^L, d^L\mu_H) \]
CS operators in LQG

Structure of LQG

\[ M = \Sigma \times \mathbb{R} \]. Hamiltonian formalism: \( g_{\mu\nu} \to (q_{ab}, p^{ab}) \), canonical ADM variables.

Symplectic transformation (to extended \( \Gamma \)) \( \Rightarrow \) canonical AB variables, \((A^a_i(x), E^a_i(x))\).

- \( A^a_i(x) \) is smeared on 1-dim submanifolds \( l \subset \Sigma \) called **links**: \( h_l(A) \in SU(2) \)
- \( E^a_i(x) \) is smeared on 2-dim submanifolds \( S \subset \Sigma \) called **surfaces**: \( P_S(E) \in su_2 = \mathbb{R}^3 \)

\( \Rightarrow \) geometry is “polymerized”, and lives on **graphs**, \( \gamma \). Hilbert space (of cylindrical functions):

\[ \mathcal{H} = \bigoplus_{\gamma} \mathcal{H}_\gamma, \quad \mathcal{H}_\gamma = L^2(SU(2)^L, d^L \mu_H) \]

Basis of spin-networks for \( \mathcal{H}_\gamma \):

\[ \psi_{j,\vec{m},\vec{n}}(\vec{g}) \equiv \langle \vec{g} | \vec{j}, \vec{m}, \vec{n} \rangle = \prod_{l=1}^L \sqrt{d_{j_l} D^{(j_l)}_{m_l n_l}(g_l)} \]
No explicit constriction of a family of CS's on $H$. The best we can do is CS's on $H^\gamma$: given $(\vec{g}_0, \vec{p}_0) \in \Gamma^\gamma = (SU(2) \times \mathbb{R}^3)_{L}$, it is

$$\psi(\vec{g}_0, \vec{p}_0) (\vec{g}) \equiv \langle \vec{g}|\vec{g}_0,\vec{p}_0 \rangle = \prod_l \sum_j d_j l e^{-t_j l (j_l + 1)/2} \chi(j_l) (g_0 l e p_0 l \cdot \sigma/2)^{-1 l}$$

(also called "heat kernel coherent states").

Resolution of identity

$$\int_{SL(2, \mathbb{C})} \mathcal{L} d\mu(\vec{g}, \vec{p}) |\vec{g},\vec{p}\rangle \langle \vec{g},\vec{p}| = \hat{I}^\gamma$$

with the measure

$$d\mu(\vec{g}, \vec{p}) = \prod_l d\mu(g_l, p_l), d\mu(g, p) \sim d\mu_H(g) d^3p \sinh|p||e^{-|p|^2/2}$$

[Thiemann, O Winkler (2001)]
CS operators in LQG

No explicit constriction of a family of CS’s on $\mathcal{H}$. The best we can do is CS’s on $\mathcal{H}_\gamma$: given $(\vec{g}_0, \vec{p}_0) \in \Gamma_\gamma = (SU(2) \times \mathbb{R}^3)^L$, it is

$$
\psi_{(\vec{g}_0, \vec{p}_0)}(\vec{g}) \equiv \langle \vec{g} | \vec{g}_0, \vec{p}_0 \rangle = \prod_l \sum_{j_l} d_{j_l} e^{-tj_l(j_l+1)/2} \chi(j_l)(g_{0l} e^{p_{0l} \cdot \sigma / 2} g_l^{-1})
$$

(also called “heat kernel coherent states”.)
No explicit constriction of a family of CS's on $\mathcal{H}$. The best we can do is CS's on $\mathcal{H}_\gamma$: given $(\tilde{g}_0, \tilde{\rho}_0) \in \Gamma_\gamma = (SU(2) \times \mathbb{R}^3)^L$, it is

$$\psi_{(\tilde{g}_0, \tilde{\rho}_0)}(\tilde{\mathbf{g}}) \equiv \langle \tilde{\mathbf{g}} | \tilde{g}_0, \tilde{\rho}_0 \rangle = \prod_l \sum_{j_l} d_{j_l} \ e^{-t j_l(j_l+1)/2} \chi^{(j_l)}(g_{0l} \ e^{\rho_{0l} \cdot \sigma} / 2 \ g_l^{-1})$$

(also called “heat kernel coherent states”.)

Resolution of identity

$$\int_{SL(2, \mathbb{C})^L} d\mu(\tilde{g}, \tilde{\rho}) \ | \tilde{\mathbf{g}}, \tilde{\mathbf{p}} \rangle \langle \tilde{\mathbf{g}}, \tilde{\mathbf{p}} | = \hat{1}_\gamma$$

with the measure [T Thiemann, O Winkler (2001)]

$$d\mu(\tilde{g}, \tilde{\rho}) = \prod_l d\mu(g_l, \rho_l), \quad d\mu(g, \rho) \sim d\mu_H(g) \ d^3 \rho \ \frac{\sinh |\rho|}{|\rho|} \ e^{-|\rho|^2/t}$$
Holonomy and Flux
Holonomy and Flux

\( j \)-representation of phase space function \( g \):

\[
\hat{A}_{D_{mn}}^{(j)} = \int d\mu(g,p) \ D_{mn}^{(j)}(g) \ | \ g,p \rangle \langle g,p |
\]

Matrix elements:

\[
\langle j_1, m_1, n_1 | \hat{A}_{D_{mn}}^{(j)} | j_2, m_2, n_2 \rangle = D_t(j, j_1, j_2) \langle j_1, m_1, n_1 | \hat{D}_{mn}^{(j)} | j_2, m_2, n_2 \rangle
\]
Holonomy and Flux

\( j \)-representation of phase space function \( g \):

\[
\hat{A}_{D_{mn}}^{(j)} = \int d\mu(g, p) \, D_{mn}^{(j)}(g) \, |g, p \rangle \langle g, p |
\]

Matrix elements:

\[
\langle j_1, m_1, n_1 | \hat{A}_{D_{mn}}^{(j)} | j_2, m_2, n_2 \rangle = D_{t}(j_1, j_2) \, \langle j_1, m_1, n_1 | \hat{D}_{mn}^{(j)} | j_2, m_2, n_2 \rangle
\]

Left-invariant vector field along \( p^m \):

\[
\hat{A}_{L m} = \int d\mu(g, p) \, p^m \, |g, p \rangle \langle g, p |
\]

Matrix elements:

\[
\langle j_1, m_1, n_1 | \hat{A}_{L m} | j_2, m_2, n_2 \rangle = E_{t}(j_1) \, \langle j_1, m_1, n_1 | \hat{L}^m | j_2, m_2, n_2 \rangle
\]
Area
Area

\[ \hat{A}_{\sqrt{L^i L^i}} = \int d\mu(g, p) \, |p| \langle g, p \rangle | g, p \rangle \]
Area

\[ \hat{A}_{\sqrt{L^i L^i}} = \int d\mu(g, p) \, |p| \, |g, p \rangle \langle g, p| \]

Matrix elements:

\[ \langle j_1, m_1, n_1 | \hat{A}_{\sqrt{L^i L^i}} | j_2, m_2, n_2 \rangle = \delta_{j_1 j_2} \delta_{m_1 m_2} \delta_{n_1 n_2} \left[ \left( \frac{1}{d_{j_1}} + \frac{d_{j_1} t}{2} \right) \text{erf} \left( \frac{\sqrt{t} d_{j_1}}{2} \right) + \sqrt{\frac{t}{\pi}} e^{-\left( j_1 (j_1 + 1) + 1/4 \right) t} \right] \]
Area

\[ \hat{A}_{\sqrt{L^iL^i}} = \int d\mu(g, p) \ |p| \ g, p \rangle \langle g, p | \]

Matrix elements:

\[ \langle j_1, m_1, n_1 | \hat{A}_{\sqrt{L^iL^i}} | j_2, m_2, n_2 \rangle = \delta_{j_1 j_2} \delta_{m_1 m_2} \delta_{n_1 n_2} \left[ \left( \frac{1}{d_{j_1}} + \frac{d_{j_1} t}{2} \right) \text{erf} \left( \frac{\sqrt{t}d_{j_1}}{2} \right) + \sqrt{\frac{t}{\pi}} e^{-\left(j_1(j_1 + 1) + 1/4\right)t} \right] \]
Volume (1)
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \hat{J}^i_l \hat{J}^j_{l'} \hat{J}^k_{l''}} \]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \hat{J}_l^i \hat{J}_{l'}^j \hat{J}_{l''}^k} \]

⇒ Need to diagonalize \( \sum_{l,l',l''} \epsilon_{ijk} \hat{J}_l^i \hat{J}_{l'}^j \hat{J}_{l''}^k \) in order to define the square root operator!
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l, l', l''} \epsilon_{ijk} \hat{J}_l^{i} \hat{J}_{l'}^{j} \hat{J}_{l''}^{k}} \]

⇒ Need to diagonalize \( \sum_{l, l', l''} \epsilon_{ijk} \hat{J}_l^{i} \hat{J}_{l'}^{j} \hat{J}_{l''}^{k} \) in order to define the square root operator!

The CS solution

\[ \hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p}|, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l, l', l''} \epsilon_{ijk} p_l^i p_{l'}^j p_{l''}^k} \]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[
\hat{V}_N \sim \sqrt{\sum_{l, l', l''} \epsilon_{ijk} \hat{J}_l^i \hat{J}_{l'}^j \hat{J}_{l''}^k}
\]

⇒ Need to diagonalize \(\sum_{l, l', l''} \epsilon_{ijk} \hat{J}_l^i \hat{J}_{l'}^j \hat{J}_{l''}^k\) in order to define the square root operator!

The CS solution

\[
\hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l, l', l''} \epsilon_{ijk} \ p_l^i \ p_{l'}^j \ p_{l''}^k}
\]

\[
\langle \vec{j}, \vec{m}, \vec{n} | A_{V_N} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \\
\sim \int d\mu(\vec{g}) \ \int d\nu(\vec{p}) \ V_N(\vec{p}) D_{\vec{m} \vec{n}}^{\vec{j}}(\vec{g}e^{\vec{p} \cdot \sigma / 2}) \overline{D_{\vec{m}' \vec{n}'}^{\vec{j}'}(\vec{g}e^{\vec{p} \cdot \sigma / 2})}
\]
Problem with canonical volume operator \cite{Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...}

\[ \hat{V}_N \sim \sqrt{\sum_{l, l', l'' \text{at } N} \epsilon_{ijk} \hat{J}_i^j \hat{J}_i^{j'} \hat{J}_i^{k'}} \]

\( \Rightarrow \) Need to diagonalize \( \sum_{l, l', l''} \epsilon_{ijk} \hat{J}_i^j \hat{J}_i^{j'} \hat{J}_i^{k'} \) in order to define the square root operator!

The CS solution

\[ \hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \, V_N(\vec{p}) \mid \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} \mid, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l, l', l'' \text{at } N} \epsilon_{ijk} \, p_i^j \, p_i^{j'} \, p_i^{k'}} \]

\[ \langle \vec{j}, \vec{m}, \vec{n} | A_{V_N} | \vec{j'}, \vec{m'}, \vec{n'} \rangle = \int d\mu(\vec{g}, \vec{p}) \, V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j'}, \vec{m'}, \vec{n'} \rangle \sim \int d\mu(\vec{g}) \int d\nu(\vec{p}) \, V_N(\vec{p}) D_{\vec{m} \vec{m}'}(\vec{g}) \, D_{\vec{m} \vec{n}'}(\vec{g}) \left( e^{\vec{p} \cdot \sigma / 2} \right) D_{\vec{n} \vec{m}'}(\vec{g}) \, D_{\vec{n} \vec{n}'}(\vec{g}) \left( e^{\vec{p} \cdot \sigma / 2} \right) \]
Problem with canonical volume operator \cite{Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...}

\[
\hat{V}_N \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \hat{J}^i_l \hat{J}^j_{l'} \hat{J}^k_{l''}}
\]

\(\Rightarrow\) Need to diagonalize \(\sum_{l,l',l''} \epsilon_{ijk} \hat{J}^i_l \hat{J}^j_{l'} \hat{J}^k_{l''}\) in order to define the square root operator!

The CS solution

\[
\hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} p^i_l p^j_{l'} p^k_{l''}}
\]

\[
\langle \vec{j}, \vec{m}, \vec{n} | A_{V_N} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \int d\mu(\vec{g}) \int d\nu(\vec{p}) \ V_N(\vec{p}) \ D^{(\vec{j})}_{\vec{m} \vec{\mu}}(\vec{g}) D^{(\vec{j})}_{\vec{\mu} \vec{\nu}}(\vec{g}) D^{(\vec{j}')}_{\vec{m}' \vec{\mu}'}(\vec{g}) D^{(\vec{j}')}_{\vec{\mu}' \vec{\nu}'}(\vec{g}) (e^{\vec{p} \cdot \sigma / 2})
\]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l''} \varepsilon_{ijk} \hat{J}_j^l \hat{J}_j^{l'} \hat{J}_j^{l''}} } \]

\[ \Rightarrow \text{Need to diagonalize} \sum_{l,l',l''} \varepsilon_{ijk} \hat{J}_j^l \hat{J}_j^{l'} \hat{J}_j^{l''} \text{ in order to define the square root operator!} \]

The CS solution

\[ \hat{A}_{VN} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l,l',l''} \varepsilon_{ijk} \ p_j^l \ p_j^{l'} \ p_j^{l''}} } \]

\[ \langle \vec{j}, \vec{m}, \vec{n} | A_{VN} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle = \]

\[ \sim \delta^{ij} \delta_{j'j} \delta_{\vec{m} \vec{m}'} \delta_{\vec{n} \vec{n}'} \int dv(\vec{p}) \ V_N(\vec{p}) D^{(j)}_{\vec{p}}(e^{\vec{p} \cdot \sigma / 2}) \ D^{(j')}_{\vec{p}}(e^{\vec{p} \cdot \sigma / 2}) \]

CS operators in LQG

Alesci, D, Lewandowski, Makinen – in preparation
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[
\hat{V}_N \sim \sqrt{\sum_{l,l',l'' \text{at } N} \epsilon_{ijk} \hat{J}_i^j \hat{J}_{l'}^i \hat{J}_{l''}^k}
\]

⇒ Need to diagonalize \(\sum_{l,l',l''} \epsilon_{ijk} \hat{J}_i^j \hat{J}_{l'}^i \hat{J}_{l''}^k\) in order to define the square root operator!

The CS solution

\[
\hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \times \vec{g}, \vec{p} |,
\quad V_N(\vec{p}) \sim \sqrt{\sum_{l,l',l'' \text{at } N} \epsilon_{ijk} \ p_i^j \ p_{l'}^i \ p_{l''}^k},
\]

\[
\langle \vec{j}, \vec{m}, \vec{n} | \hat{A}_{V_N} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \delta^{\vec{j}}_{\vec{j}'} \delta_{\vec{m} \vec{m}'} \int d\nu(\vec{p}) \ V_N(\vec{p}) D_{\vec{j} \vec{m} \vec{n}}^j(\vec{p} \cdot \vec{s}/2) \ D_{\vec{j} \vec{m} \vec{n}'}^{\vec{j}'}(\vec{p} \cdot \vec{s}/2)
\]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \hat{J}_i^l \hat{J}_j^{l'} \hat{J}_k^{l''}} } \]

⇒ Need to diagonalize \[ \sum_{l,l',l''} \epsilon_{ijk} \hat{J}_i^l \hat{J}_j^{l'} \hat{J}_k^{l''} \] in order to define the square root operator!

The CS solution

\[ \hat{A}_{VN} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \ p_i^l \ p_j^{l'} \ p_k^{l''}} } \]

\[ \langle \vec{j}, \vec{m}, \vec{n} | A_{VN} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \]

\[ \sim \delta^{\vec{j} \vec{j}'} \delta_{\vec{m} \vec{m}'} \int d\nu(\vec{p}) \ V_N(\vec{p}) \ D_{i\vec{m}}^{\vec{j}}(\vec{e} \cdot \vec{p}/2) \ D_{\vec{m}' \vec{n}'}^{\vec{j}'}(\vec{e} \cdot \vec{p}/2) \]

Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[
\hat{V}_N \sim \sqrt{\left| \sum_{l, l', l'' \atop at N} \epsilon_{ijk} \hat{J}_i \hat{J}_j \hat{J}_k \right|} 
\]

⇒ Need to diagonalize \( \sum_{l, l', l''} \epsilon_{ijk} \hat{J}_i \hat{J}_j \hat{J}_k \) in order to define the square root operator!

The CS solution

\[
\hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \; V_N(\vec{p}) \; | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\left| \sum_{l, l', l'' \atop at N} \epsilon_{ijk} \; p_i \; p_j \; p_k \right|} 
\]

\[
\langle \vec{j}, \vec{m}, \vec{n} | A_{V_N} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \; V_N(\vec{p}) \; \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \delta^{\vec{j} \vec{j}'} \delta_{\vec{m} \vec{m}'} \int d\nu(\vec{p}) \; V_N(\vec{p}) \; D^{(\vec{j})}_{\vec{n} \vec{n}'}(e^{\vec{p} \cdot \sigma})
\]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l'' \text{at } N} \epsilon_{ijk} \hat{\mathcal{J}}^i_{l} \hat{\mathcal{J}}^j_{l'} \hat{\mathcal{J}}^k_{l''}} \]

⇒ Need to diagonalize \( \sum_{l,l',l''} \epsilon_{ijk} \hat{\mathcal{J}}^i_{l} \hat{\mathcal{J}}^j_{l'} \hat{\mathcal{J}}^k_{l''} \) in order to define the square root operator!

The CS solution

\[ \hat{A}_{VN} = \int d\mu(\tilde{g}, \tilde{p}) \ V_N(\tilde{p}) \ | \tilde{g}, \tilde{p} \rangle \langle \tilde{g}, \tilde{p} |, \quad V_N(\tilde{p}) \sim \sqrt{\sum_{l,l',l'' \text{at } N} \epsilon_{ijk} \ p^i_l \ p^j_{l'} \ p^k_{l''}} \]

\[ \langle \tilde{j}, \tilde{m}, \tilde{n} | A_{VN} | \tilde{j}', \tilde{m}', \tilde{n}' \rangle = \int d\mu(\tilde{g}, \tilde{p}) \ V_N(\tilde{p}) \ \langle \tilde{j}, \tilde{m}, \tilde{n} | \tilde{g}, \tilde{p} \rangle \langle \tilde{g}, \tilde{p} | \tilde{j}', \tilde{m}', \tilde{n}' \rangle \sim \]

\[ \sim \delta_{\tilde{j} \tilde{j}'} \delta_{\tilde{m} \tilde{m}'} \int d\nu(\tilde{p}) \ V_N(\tilde{p}) D_{\tilde{n}' \tilde{n}}^{(j)}(e^{\tilde{p} \cdot \sigma}) \]
Volume (1)

Problem with canonical volume operator [Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]

\[ \hat{V}_N \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \hat{J}_i \hat{J}_j \hat{J}_k,} \]

⇒ Need to diagonalize \( \sum_{l,l',l''} \epsilon_{ijk} \hat{J}_i \hat{J}_j \hat{J}_k, \) in order to define the square root operator!

The CS solution

\[ \hat{A}_{V_N} = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \ | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} |, \quad V_N(\vec{p}) \sim \sqrt{\sum_{l,l',l''} \epsilon_{ijk} \ p_i \ p_j \ p_k}, \]

\[ \langle \vec{j}, \vec{m}, \vec{n} | \hat{A}_{V_N} | \vec{j}', \vec{m}', \vec{n}' \rangle = \int d\mu(\vec{g}, \vec{p}) \ V_N(\vec{p}) \langle \vec{j}, \vec{m}, \vec{n} | \vec{g}, \vec{p} \rangle \langle \vec{g}, \vec{p} | \vec{j}', \vec{m}', \vec{n}' \rangle \sim \]

\[ \sim \delta^{\vec{j}}_{\vec{j}'} \delta_{\vec{m}'} \delta_{\vec{n}'} \ A^{(\vec{j})}_{\vec{m} \vec{n}} \]
Thus,

\[ \hat{A}_{V_N}(\vec{j}, \vec{m}, \vec{n}) = \sum_{\vec{n}'} A^{(\vec{j})}_{\vec{n} \vec{n}'} (\vec{j}, \vec{m}, \vec{n}') \]
Thus,

\[ \hat{A}_{VN}(\vec{j}, \vec{m}, \vec{n}) = \sum_{\vec{n}'} A^{(j)}_{\vec{n} \vec{n}'} |\vec{j}, \vec{m}, \vec{n}'\rangle \]

Consider a gauge-invariant at node \( N \):

\[ |\vec{j}, \vec{m}, \iota^{(k)}\rangle = \sum_{\vec{n}} \iota^{(k)}_{\vec{n}} |\vec{j}, \vec{m}, \vec{n}\rangle \]
Volume (2)

Thus,

$$\hat{A}_{VN} |\vec{j}, \vec{m}, \vec{n}\rangle = \sum_{\vec{n}'} A^{(\vec{j})}_{\vec{n} \vec{n}'} |\vec{j}, \vec{m}, \vec{n}'\rangle$$

Consider a gauge-invariant at node $N$:

$$|\vec{j}', \vec{m}, \iota^{(k)}\rangle = \sum_{\vec{n}} \iota^{(k)}_{\vec{n}} |\vec{j}, \vec{m}, \vec{n}\rangle$$

Then

$$\hat{A}_{VN} |\vec{j}', \vec{m}, \iota^{(k)}\rangle = \sum_{\vec{n}} \iota^{(k)}_{\vec{n}} \sum_{\vec{n}'} A^{(\vec{j})}_{\vec{n} \vec{n}'} |\vec{j}', \vec{m}, \vec{n}'\rangle$$
Thus,

\[ \hat{A}_{VN} \langle \vec{j}, \vec{m}, \vec{n} \rangle = \sum_{\vec{n}'} A_{\vec{n} \vec{n}'}^{(\vec{j})} \langle \vec{j}, \vec{m}, \vec{n}' \rangle \]

Consider a gauge-invariant at node \( N \):

\[ \langle \vec{j}, \vec{m}, \iota^{(k)} \rangle = \sum_{\vec{n}} l_{\vec{n}}^{(k)} \langle \vec{j}, \vec{m}, \vec{n} \rangle \]

Then

\[ \hat{A}_{VN} \langle \vec{j}, \vec{m}, \iota^{(k)} \rangle = \sum_{\vec{n}'} \sum_{\vec{n}} l_{\vec{n}}^{(k)} A_{\vec{n} \vec{n}'}^{(\vec{j})} \langle \vec{j}, \vec{m}, \vec{n}' \rangle \]
Thus,

$$\hat{A}_{V_N} |j, m, n, \nu(\kappa)\rangle = \sum_{\tilde{n}} A_{\tilde{n} n'}^{(j)} |j, m, n', \nu\rangle$$

Consider a gauge-invariant at node $N$:

$$|j, m, \nu(\kappa)\rangle = \sum_{\tilde{n}} \nu_{\tilde{n}}^{(k)} |j, m, \tilde{n}\rangle$$

Then

$$\hat{A}_{V_N} |j, m, \nu(\kappa)\rangle = \sum_{\tilde{n}'} \nu_{\tilde{n}'}^{(\kappa')} |j, m, \tilde{n}'\rangle$$
Thus,
\[ \hat{A}_{V_N} |\vec{j}, \vec{m}, \vec{n}\rangle = \sum_{\vec{n}'} A_{\vec{n} \vec{n}'}^{(\vec{j})} |\vec{j}, \vec{m}, \vec{n}'\rangle \]

Consider a gauge-invariant at node \( N \):
\[ |\vec{j}, \vec{m}, \iota^{(k)}\rangle = \sum_{\vec{n}} \iota_{\vec{n}}^{(k)} |\vec{j}, \vec{m}, \vec{n}\rangle \]

Then
\[ \hat{A}_{V_N} |\vec{j}, \vec{m}, \iota^{(k)}\rangle = \sum_{\vec{n}'} \iota^{(k')}_{\vec{n}'} |\vec{j}, \vec{m}, \vec{n}'\rangle \]

⇒ Don’t need to diagonalize \( A_{\vec{n} \vec{n}'}^{(\vec{j})} \) for the action of \( \hat{A}_{V_N} \) on gauge-invariant spin-networks!
coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions
conclusions

Take-home messages:

⇒ constructing operators? there exists a procedure alternative to the canonical one
⇒ operators thus built have automatically the correct semiclassical limit
⇒ CS "quantization" of homogeneous isotropic cosmology leads to singularity resolution
⇒ anisotropic Bianchi I is under study
⇒ can be useful in LQG to construct operators which are non-polynomial functions of the fundamental variables
⇒ volume is under study
⇒ But don't be too happy:
⇒ classical Poisson algebra is lost
⇒ applications to LQG are limited to fixed graph, until CS's on the full H are found
conclusions

Take-home messages:

⇒ constructing operators? there exists a procedure alternative to the canonical one

• anisotropic Bianchi I is under study

⇒ can be useful in LQG to construct operators which are non-polynomial functions of the fundamental variables

• volume is under study

But don't be too happy:

⇒ classical Poisson algebra is lost

⇒ applications to LQG are limited to fixed graph, until CS's on the full $H$ are found
Take-home messages:
⇒ constructing operators? there exists a procedure alternative to the canonical one
⇒ operators thus built have automatically the correct semiclassical limit
Take-home messages:

⇒ constructing operators? there exists a procedure alternative to the canonical one

⇒ operators thus built have automatically the correct semiclassical limit

⇒ CS “quantization” of homogeneous isotropic cosmology leads to singularity resolution
  • anisotropic Bianchi I is under study

But don’t be too happy:

⇒ classical Poisson algebra is lost

⇒ applications to LQG are limited to fixed graph, until CS’s on the full $H$ are found
conclusions

Take-home messages:

⇒ constructing operators? there exists a procedure alternative to the canonical one

⇒ operators thus built have automatically the correct semiclassical limit

⇒ CS “quantization” of homogeneous isotropic cosmology leads to singularity resolution
  • anisotropic Bianchi I is under study

⇒ can be useful in LQG to construct operators which are non-polynomial functions of the fundamental variables
  • volume is under study

But don’t be too happy:
conclusions

Take-home messages:
⇒ constructing operators? there exists a procedure alternative to the canonical one
⇒ operators thus built have automatically the correct \textit{semiclassical limit}
⇒ CS “quantization” of homogeneous isotropic cosmology leads to \textit{singularity resolution}
  • anisotropic Bianchi I is under study
⇒ can be useful in LQG to construct operators which are \textit{non-polynomial} functions of the fundamental variables
  • volume is under study

But don’t be too happy:
⇒ classical Poisson algebra is lost
conclusions

Take-home messages:
⇒ constructing operators? there exists a procedure alternative to the canonical one
⇒ operators thus built have automatically the correct semiclassical limit
⇒ CS “quantization” of homogeneous isotropic cosmology leads to singularity resolution
  • anisotropic Bianchi I is under study
⇒ can be useful in LQG to construct operators which are non-polynomial functions of the fundamental variables
  • volume is under study

But don’t be too happy:
⇒ classical Poisson algebra is lost
⇒ applications to LQG are limited to fixed graph, until CS’s on the full $\mathcal{H}$ are found
merci beaucoup!