Coherent State Operators in Cosmology and Gravity

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H Bergeron, AD, J P Gazeau, P Malkiewicz - Phys. Rev. D **89**, 083522 (2014) [arXiv:1305.0653]
E Alesci, AD, J Lewandowski, I Makinen - *in preparation*

outline

coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions

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Many definitions of a family of *coherent states* (CS). But the bare minimum is:

- peakedness
- resolution of identity

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Peakedness: Each CS in the family is labelled by a point $(q, p) \in \Gamma$. It is “peaked” on it, i.e.

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In general, CS's in $\{|q,p\rangle\}$ are not orthogonal, $\langle q,p|q',p'\rangle \neq \delta_{qq'}\delta_{pp'}$. E.g., for Gaussian CS's

$$\langle x|q,p\rangle = N\,e^{-(x-(q-ip))^2/2t}$$

it is

$$|\langle q,p|q',p'\rangle| \sim \exp\left(-\frac{1}{4t}\left[(q-q')^2+(p-p')^2\right]\right) \xrightarrow{t\to 0} \delta_{qq'}\delta_{pp'}$$

coherent state “quantization”

Resolution of Identity: there exists a measure $\mathrm{d}\mu(q,p)$ on Γ such that

$$\int \mathrm{d}\mu(q,p)\, |q,p\rangle\langle q,p| = \hat{I}$$

This ensures that the family $\{|q,p\rangle\}$ is a (generally overcomplete) basis for the Hilbert space.

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$\Rightarrow$ Extend the resolution of identity to *any* function on Γ:

$$1 \quad \longrightarrow \quad \hat{A}_1 := \int \mathrm{d}\mu(q,p)\, 1\, |q,p\rangle\langle q,p|$$

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Inbuilt semiclassical limit:

$$\langle q,p|\hat{A}_F|q,p\rangle = \int \mathrm{d}\mu(q',p')\, F(q',p')\, \left|\langle q,p|q',p'\rangle\right|^2 = F(q,p) + O(t)$$

canonical vs coherent

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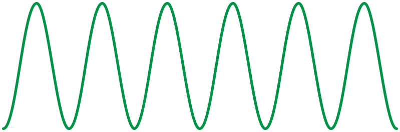
$$x \;,\; -i\hbar\frac{d}{dx} \qquad\qquad \hat{a} \;,\; \hat{a}^\dagger$$

• ,



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For canonical variables and some special symmetries, the Poisson algebra carries over to the quantum level:

$$[\hat{Q}, \hat{P}] = i\hbar\,\hat{I} \qquad \Rightarrow \qquad \left[\hat{J}_1,\ \hat{J}_2\right] = i\hbar\,\hat{J}_3$$

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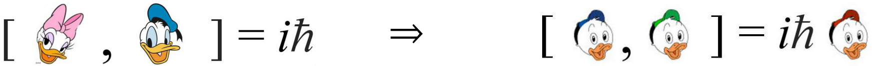

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So we should not be too sad if CS operators do not respect any canonical relation:

$$[\hat{A}_F, \hat{A}_G] \neq i\hbar \, \hat{A}_{\{F,G\}}$$

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Classical Theory

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Consider homogeneous isotropic cosmology with perfect fluid ($p = w\rho$): with a proper choice of variables **[B F Schutz (1970, 1971)]**

- fluid $(T, p_T) \in \mathbb{R} \times \mathbb{R}_+$
- gravitation $(q, p) \in \mathbb{R}_+ \times \mathbb{R}$

the Hamiltonian constraint in comoving coordinates

$$H = -p^2 - k\,\beta(w)\,q^{\mu(w)} + p_T$$

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⇒ Physical Hamiltonian on gravitational phase space:

$$h(q, p) = p^2 + k\,\beta(w)\,q^{\mu(w)}$$

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In our case, the structure of $\Gamma = \mathbb{R}_+ \times \mathbb{R}$ suggests the use of affine group:

$$\langle x|q,p\rangle = \left(U(q,p)\,\psi_0\right)(x) = \frac{e^{ipx}}{\sqrt{q}}\,\psi_0(x/q)$$

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We can construct CS operators! In particular,

$$\hat{A}_{p^2} = \hat{P}^2 + K(\psi_0)\,\hat{Q}^{-2}, \qquad K(\psi_0) := \int_0^\infty \frac{\mathrm{d}y}{c}\,y\,|\psi_0'(y)|^2$$

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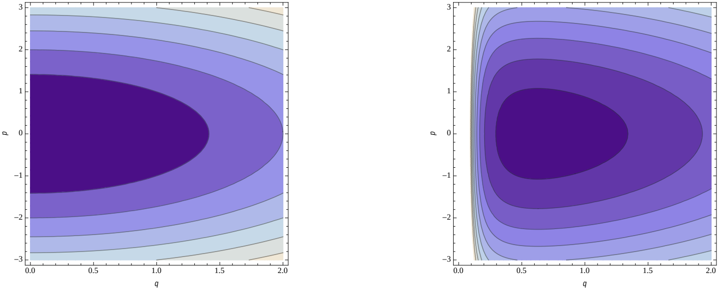
$\Rightarrow$ integral curves of h_{eff} consistently avoid the $q=0$ region.



(Integral curves of h and h_{eff} respectively, for $k=1$.)

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Basis of spin-networks for $\mathcal{H}_\gamma$:

$$\psi_{\vec{j},\vec{m},\vec{n}}(\vec{g}) \;\equiv\; \langle\, \vec{g} \,|\, \vec{j}, \vec{m}, \vec{n} \,\rangle \;=\; \prod_{l=1}^{L} \sqrt{d_{j_l}}\; \overline{D^{(j_l)}_{m_l n_l}(g_l)}$$

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No explicit constriction of a family of CS's on $\mathcal{H}$. The best we can do is CS's on $\mathcal{H}_\gamma$: given $(\vec{g}_0, \vec{\boldsymbol{p}}_0) \in \Gamma_\gamma = (SU(2) \times \mathbb{R}^3)^L$, it is

$$\psi_{(\vec{g}_0,\vec{\boldsymbol{p}}_0)}(\vec{g}) \equiv \langle\, \vec{g} \mid \vec{g}_0, \vec{\boldsymbol{p}}_0 \,\rangle = \prod_l \sum_{j_l} d_{j_l}\, e^{-t\, j_l(j_l+1)/2}\, \chi^{(j_l)}(g_{0l}\, e^{\boldsymbol{p}_{0l}\cdot\boldsymbol{\sigma}/2}\, g_l^{-1})$$

(also called “heat kernel coherent states”.)

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Resolution of identity

$$\int_{SL(2,\mathbb{C})^L} \mathrm{d}\mu(\vec{g},\vec{\boldsymbol{p}}) \mid \vec{g},\vec{\boldsymbol{p}}\,\rangle\langle\, \vec{g},\vec{\boldsymbol{p}} \mid \;=\; \hat{I}_\gamma$$

with the measure **[T Thiemann, O Winkler (2001)]**

$$\mathrm{d}\mu(\vec{g},\vec{\boldsymbol{p}}) = \prod_l \mathrm{d}\mu(g_l,\boldsymbol{p}_l), \qquad \mathrm{d}\mu(g,\boldsymbol{p}) \sim \mathrm{d}\mu_H(g)\; \mathrm{d}^3p\; \frac{\sinh|\boldsymbol{p}|}{|\boldsymbol{p}|}\, e^{-|\boldsymbol{p}|^2/t}$$

Holonomy and Flux

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j-representation of phase space function g:

$$\hat{A}_{D^{(j)}_{mn}} = \int \mathrm{d}\mu(g, \boldsymbol{p})\; D^{(j)}_{mn}(g)\, |\, g, \boldsymbol{p}\, \rangle\langle\, g, \boldsymbol{p}\, |$$

Matrix elements:

$$\langle j_1, m_1, n_1 | \hat{A}_{D^{(j)}_{mn}} | j_2, m_2, n_2 \rangle = D_t(j, j_1, j_2)\; \langle j_1, m_1, n_1 | \hat{D}^{(j)}_{mn} | j_2, m_2, n_2 \rangle$$

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Left-invariant vector field along p^m:

$$\hat{A}_{L^m} = \int \mathrm{d}\mu(g, \boldsymbol{p})\; p^m \mid g, \boldsymbol{p} \,\rangle\langle\, g, \boldsymbol{p} \mid$$

Matrix elements:

$$\langle j_1, m_1, n_1 | \hat{A}_{L^m} | j_2, m_2, n_2 \rangle = E_t(j_1)\; \langle j_1, m_1, n_1 | \hat{L}^m | j_2, m_2, n_2 \rangle$$

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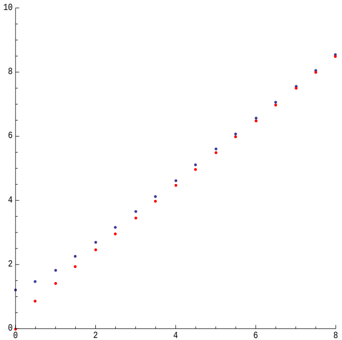

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Problem with canonical volume operator **[Rovelli, Smolin, Ashtekar, Lewandowski, Brunnemann, Thiemann, and many others...]**

$$\hat{V}_N \sim \sqrt{\left| \sum_{l,l',l'' \text{at } N} \epsilon_{ijk}\, \hat{\mathcal{J}}^i_l\, \hat{\mathcal{J}}^j_{l'} \hat{\mathcal{J}}^k_{l''} \right|}$$

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The CS solution

$$\hat{A}_{V_N} = \int \mathrm{d}\mu(\vec{g}, \vec{\boldsymbol{p}}) \; V_N(\vec{\boldsymbol{p}}) \mid \vec{g}, \vec{\boldsymbol{p}} \,\rangle\langle\, \vec{g}, \vec{\boldsymbol{p}} \mid, \qquad V_N(\vec{\boldsymbol{p}}) \sim \sqrt{\left| \sum_{l,l',l'' \text{at } N} \epsilon_{ijk} \; p^i_l \; p^j_{l'} p^k_{l''} \right|}$$

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CS operators in LQG

Alesci, D, Lewandowski, Makinen – *in preparation*

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$\Rightarrow$ Don't need to diagonalize $A^{(\vec{j})}_{\vec{n}\,\vec{n}'}$ for the action of $\hat{A}_{V_N}$ on gauge-invariant spin-networks!

outline

coherent state “quantization”

example 1: quantum cosmology

example 2: quantum gravity

conclusions

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Take-home messages:

conclusions

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⇒ applications to LQG are limited to fixed graph, until CS’s on the full $\mathcal{H}$ are found merci beaucoup!