# From Kerr-Newman Black Hole to Spinning Particle: Where is There Hidden the Dirac Equation?

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Based on:

***A.B., Regularized Kerr-Newman solution as Gravitating Soliton, J.Phys.A, 43, 392001 (2010).***

***A.B., Emergence of the Dirac Equation in the Solitonic Source of the Kerr Spinning Particle, [arXiv:1404.5947].***

## KERR-NEWMAN SPINNING PARTICLE AS A DRESSED ELECTRON.

**Black holes as elementary particles**[ G.'t Hooft (1990), A. Sen (1995), C.F.E. Holzhey and F. Wilczek (1992), A.Salam and J. Strathdee (1976)].

**The experimentally observable parameters of the electron (mass $m$, spin $J$, charge $e$ and magnetic moment $\mu$) determine that gravitational and electromagnetic fields of the electron are to be described by the Kerr-Newman (KN) black hole solution!**

**Spin of electron is extremely high, $a = J/m >> m$ ($a/m \approx 10^{44}$ ), and the black hole horizons disappear, which corresponds to OVER-EXTREMAL KN solution.**

**Background of the over-extremal KN solution contains topological defect - NAKED SINGULAR RING OF THE COMPTON RADIUS , which is branch line of space-time forming a "door" to a mirror world resulting in *TWO-SHEETED space-time!***

**THE CONFLICT GRAVITY AND QUANTUM THEORY STARTED ALREADY ON THE COMPTON SCALE – much before the Planck scale!**

## NAKED KERR SINGULAR RING OF THE COMPTON RADIUS.

***Quantum theory requires normal FLAT space.***

***To remove the conflict, the Kerr space should be REGULARIZED, or reduced to flat one!***

**This requirement determines UNAMBIGUOUSLY *the structure of soliton source of the KN solution.***

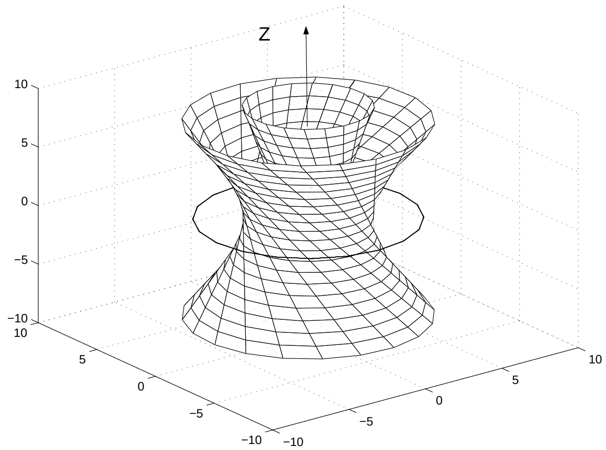
**The SOURCE takes the form of a BAG (false vacuum bubble), *similar to MIT-bag and SLAC-bag models of the extended hadrons.***

**We show that this bag has the Compton size, and applying this model to an electron, we shall consider it as a model of *dressed* electron.**

***We discuss emergence of the Dirac equation in the bag-like source of the KN solution.***

# REAL structure of the Kerr-Newman solution:

**Metric** $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}$, **and electromagnetic vector potential** $A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu$ **are collinear with** $k^\mu$.




**The Kerr singular ring is a branch line of space leading to TWOSHEETED Kerr space! Kerr congruence $k^\mu$ is in-going on the positive sheet and out-going on the negative one. The bag-like source covers the "door" to negative sheet and removes twosheetedness!**

**Kerr congruence is controlled by the KERR THEOREM:**

***Geodesic and Shear-free congruences are obtained as analytic solutions of the equation*** $F(T^a) = 0$ , ***where*** $F$ ***is a holomorphic function of the projective twistor coordinates in*** $\boldsymbol{CP}^3$, $\qquad T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}$.

# PECULIARITIES OF MIT-BAG AND KN-BAG MODELS.

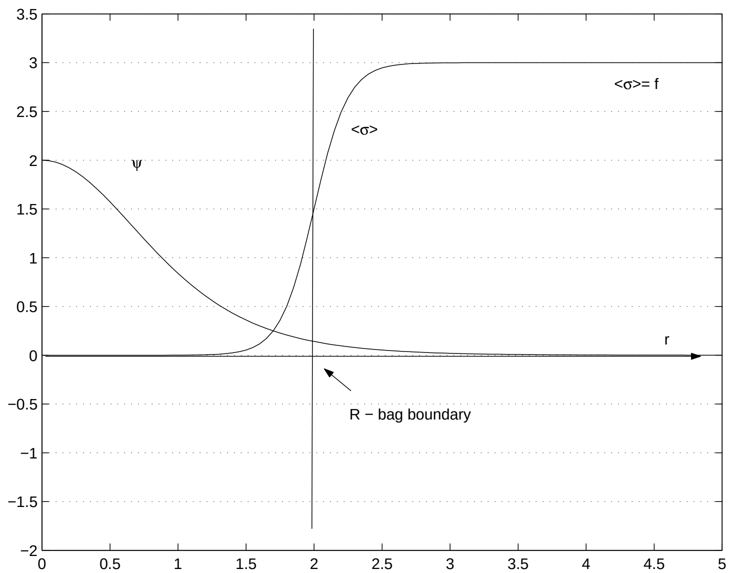



Figure 1: Illustration of the quark confinement in the bag models. Vacuum field $\sigma$ is determined by quartic potential.

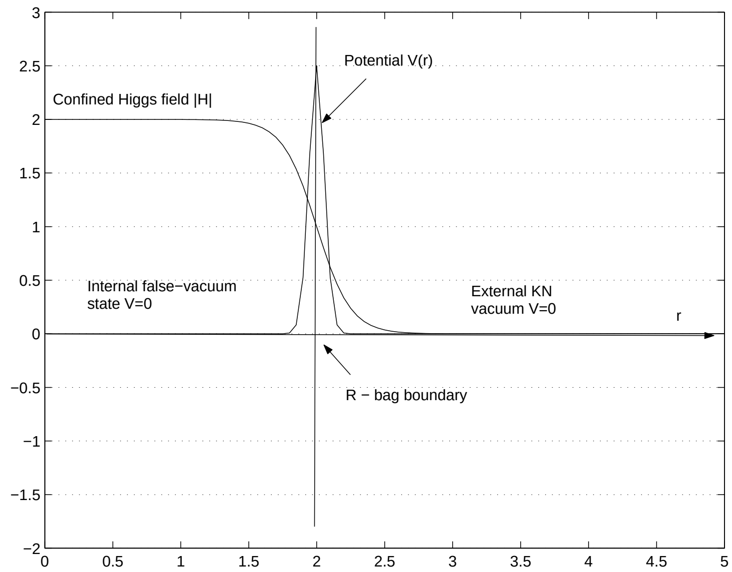



Figure 2: The KN soliton bag model (Q-ball). Potential $V(R)$ forms a narrow spike at the bag boundary. The Higgs field $H$ is confined inside the bag forming a false-vacuum state.

**In MIT- and SLAC-bag models the gauge symmetry is broken outside the bag: the standard quartic potential $V(r) = \lambda(|\phi|^2 - \Phi^2)^2$ is unacceptable for solitons with external gravitational and electromagnetic field!**
**KN-bag should have $V_{int} = V_{ext} = 0$, and a narrow spike at the bag-boundary.**
**Formation of such a potential requires *a few chiral fields* $\Phi^i(r), \ i = 1, 2, 3.$**

## Supersymmetric field model of phase transition.

**Triplet of the chiral fields** $\Phi^{(i)} = \{H, Z, \Sigma\}$, where $H$ is Higgs field.

**Lagrangian**

$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{3} F^{(i)}_{\mu\nu}F^{(i)\mu\nu} + \frac{1}{2}\sum_{i=1}^{3}(\mathcal{D}^{(i)}_{\mu}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^{*} + V, \tag{1}$$

**covariant derivatives** $\mathcal{D}^{(i)}_{\mu} = \nabla_{\mu} + ieA^{(i)}_{\mu}$.

**Superpotential**

$$W = \Phi^{(2)}(\Phi^{(1)}\bar{\Phi}^{(1)} - \eta^{2}) + (\Phi^{(2)} + \mu)\Phi^{(3)}\bar{\Phi}^{(3)}, \tag{2}$$

**determines the potential**

$$V(r) = \sum_{i} |\partial_i W|^2, \tag{3}$$

$\mathcal{H} \equiv \Phi^{(1)}$ **is taken as Higgs field.**

**Vacuum states** $V_{(vac)} = 0$ **are determined by the conditions** $\partial_i W = 0$. **The model yields two vacuum solutions:**

**(I) vacuum state inside the bag:** $|\mathcal{H}| = \eta\sqrt{\lambda};\ Z = -\mu;\ \Sigma = 0,$

**(II) external vacuum state:** $|\mathcal{H}| = 0;\ Z = 0;\ \Sigma = \eta.$

**The Higgs field $\mathcal{H}$ is confined inside the bag. Gauge symmetry is broken $\Rightarrow$ false vacuum state.**

**Basic equations for interaction of the electromagnetic and the Higgs field $\mathcal{H}(x) = |\mathcal{H}|e^{i\chi(x)}$ confined inside the bubble:**

$$\mathcal{D}^{(1)}_\nu \mathcal{D}^{(1)\nu} \mathcal{H} = \partial_{\mathcal{H}^*} V, \quad (4)$$

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e|\mathcal{H}|^2 (\chi,_\mu + eA_\mu). \quad (5)$$

***Peculiarities of the KN soliton model***:

**(i) the Kerr singular ring is regularized, and forms a circular string of the Compton radius $r_c \approx a$ along the sharp border of the disklike bag,**

**(ii) closed flux of the KN electromagnetic potential forms a *quantum Wilson loop* $\oint eA_\varphi d\varphi = -4\pi ma$, which results in quantization of the soliton spin**, $J = ma = n\hbar/2, \ n = 1, 2, 3, ...,$

**(iii) the Higgs condensate forms a *coherent vacuum state* oscillating with the frequency $\omega = 2m$ − oscillons, Q-balls (G.Rosen 1968, Coleman 1985).**

**Shape of the bag is unambiguously determined from the form of KN metric $g^{(KN)}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)}k_\mu k_\nu$ by the condition**

$$H_{(KN)} = mr - e^2/2 = 0 \Rightarrow r = e^2/2m \qquad (6)$$

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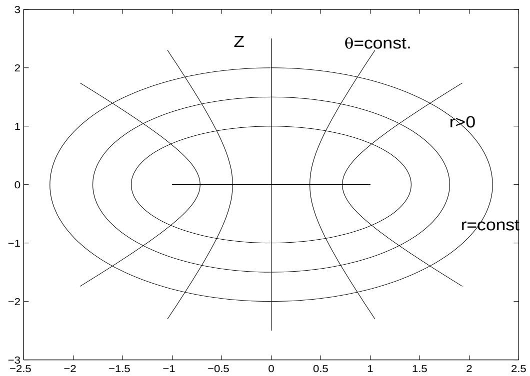


Figure 3: **Kerr's oblate spheroidal coordinates cover space-time twice.**

**The bag forms an oblate disk of the Compton radius $r_c \approx a = \frac{1}{2m}$ with the thickness of classical EM radius of electron $r_e = \frac{e^2}{2m}$, so that $r_e/r_c = \alpha \approx 137^{-1}$. The soliton bag closes the door to negative sheet of the Kerr space-time. However, the second sheet emerges again from another side.**

**Dirac: Radiation reaction of classical electron.**

**Following to Dirac and Feynman, the retarded potentials $A_{ret}$ are represented in the form**

$$A_{ret} = \frac{1}{2}[A_{ret} + A_{adv}] + \frac{1}{2}[A_{ret} - A_{adv}], \quad (7)$$

**where half-difference corresponds to radiation, and half-sum – to a self-interaction of the source.**

**For the extended KN source we connect the dynamics and mass-formation of the solitonic source with a field of advanced potentials.**

**In accord with peculiarities of the Kerr-Schild solutions, the fields $A_{ret}$ and $A_{adv}$ cannot reside on the same physical sheet, because each of this fields should be aligned with the corresponding Kerr congruence.**

**The null vector fields $k^{\mu\pm}(x)$ differ on the retarded and advanced sheets, and generate different metrics**

$$g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)}k^{\pm}_{\mu}k^{\pm}_{\nu}. \quad (8)$$

**The retarded and advanced metrics are not compatible and the corresponding fields should be positioned on separate sheets.**

**However, this problem disappears inside the bag, where the space is flat, and the both null congruences $k^{\pm}_{\mu}(x)$ are null with respect to the flat Minkowski space.**

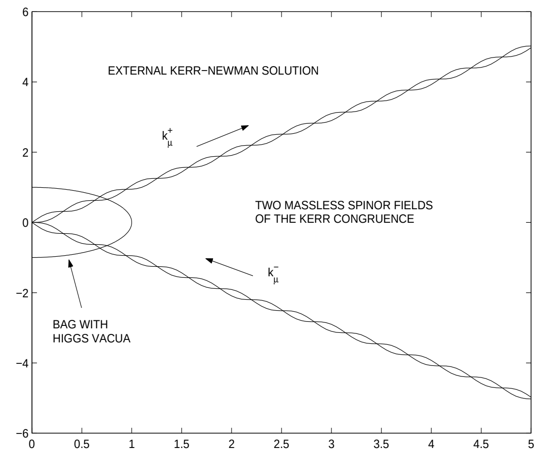


Figure 4: Dirac equation is built of two massless spinor fields $\phi_\alpha$ $\bar{\chi}^{\dot{\alpha}}$ positioned on two different sheets of the Kerr geometry covered by antipodal Kerr congruences (solutions $Y^+(x)$ and $Y^-(x) = -1/\bar{Y}^+$). Penetrating inside the bag these fields acquire Yukawa coupling which yields mass term to the Dirac equation.

**Two different null congruences determined by two conjugate solutions of *the Kerr theorem* $Y^{\pm}(x^\mu)$.**
**The Kerr theorem determines all the geodesic and *shear free* congruences as analytical solutions of the equation**

$$F(T^A) = 0, \tag{9}$$

**where $F$ is an arbitrary holomorphic function of the projective twistor variables**

$$T^A = \{Y,\ \zeta - Yv,\ u + Y\bar{\zeta}\}, \qquad A = 1, 2, 3. \tag{10}$$

**Coordinates**

$$\begin{aligned} \zeta &= (x+iy)/\sqrt{2}, \quad \bar{\zeta} = (x-iy)/\sqrt{2}, \\ u &= (z+t)/\sqrt{2}, \quad v = (z-t)/\sqrt{2} \end{aligned} \tag{11}$$

**are the null Cartesian coordinates of the Minkowski space** $x^\mu = (t, x, y, z) \in M^4$**, and parameter** $Y$ **is a projective spinor coordinate**

$$Y = \phi_1/\phi_0, \tag{12}$$

**which is equivalent to the Weyl two-component spinor** $\phi_\alpha$**.**

**Function** $F$ **for the Kerr and KN solutions may be represented in the quadratic in** $Y$ **form,**

$$F(Y, x^\mu) = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu). \tag{13}$$

**In this case, the equation (9) is explicitly solved, leading to two solutions**

$$Y^\pm(x^\mu) = (-B \mp \tilde{r})/2A, \tag{14}$$

**where** $\tilde{r} = (B^2 - 4AC)^{1/2}$**. It was shown in [?] that these two solutions are antipodally conjugate**

$$Y^+ = -1/\bar{Y}^-. \tag{15}$$

**The solutions (14) determine two Weyl spinor fields $\phi^\alpha$ and $\bar\chi_{\dot\alpha}$, which in agreement with (12) can be set in correspondence to two antipodal congruences**

$$Y^+ = \phi_1/\phi_0, \tag{16}$$

$$Y^- = \bar\chi^{\dot 1}/\bar\chi^{\dot 0}. \tag{17}$$

**In DKS formalism function $Y$ is also a projective angular coordinate $Y^+ = e^{i\phi}\tan\frac{\theta}{2}$, and the created spinor fields $\phi_\alpha$ and $\bar\chi^{\dot\alpha}$ acquire also explicit dependence dependence on $\phi$ and $\theta$. For the congruence $Y^+$ this dependence takes the form**

$$\phi_\alpha = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \tag{18}$$

**and for $Y^- = -1/\bar Y^+$, we have**

$$\bar\chi^{\dot\alpha} = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \tag{19}$$

**These massless spinor fields are connected to the left-handed and right-handed congruences, and only one of them, say "left-handed", $k_\mu^{(+)}(x)$ is "retarded" and corresponds to the external KN solution. Vector field $k_\mu^{(\pm)}(x)$ is determined in DKS formalism by the differential form**

$$k_\mu dx^\mu = P^{-1}(du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv), \tag{20}$$

**where $P = (1 + Y\bar{Y})/\sqrt{2}$ may be considered as a normalizing factor for the time-like component , $k_0^{(\pm)}(x) = 1$. Antipodal map (15) transforms the normalized field $k_\mu^{(+)}(x) = (1, \mathbf{k})$ in the field $k_\mu^{(-)}(x) = (1, -\mathbf{k})$, which retains the time-like direction and reflects the space orientation. Therefore, the spinor fields created by the Kerr theorem $\phi_\alpha$ and $\bar{\chi}^{\dot{\alpha}}$ correspond to the left out-field and right-in fields, i.e. the retarded and advanced fields correspondingly.**

**The expression (20) is indeed analog of the bilinear spinor representation of the null vector, $k^\mu = \bar{\phi}\sigma^\mu\phi$, in terms of two-component Weyl's spinor field $\phi_\alpha = \begin{pmatrix} \phi_1 \\ \phi_0 \end{pmatrix}$.**

**The Dirac equation**

$$(-i\gamma^\mu\partial_\mu + m)\Psi = 0, \tag{21}$$

**follows from Lagrangian**

$$\mathcal{L} = \frac{i}{2}(\bar{\Psi}\gamma^\mu \overrightarrow{\partial}_\mu \Psi - \bar{\Psi}\gamma^\mu \overleftarrow{\partial}_\mu \Psi) + m\bar{\Psi}\Psi, \tag{22}$$

**where $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$, and $\bar{\Psi} = \Psi^\dagger\gamma^0$ is the Dirac-conjugate spinor.**

**The free Dirac equation splits in the Weyl representation into two equations**

$$\sigma^\mu_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \tag{23}$$

**which in Standard Model are sometimes called the left-handed and right-handed electron fields. The mass therm in Lagrangian is represented by scalar** $S = -\bar{\Psi}\Psi$**. In the Weyl representation, it takes the form**

$$S = -\bar{\Psi}\Psi = -m(\bar{\phi}\chi + \bar{\chi}\phi). \qquad (24)$$

**The free electron with 4-momentum** $p$ **is described by plane wave**

$$\Psi_p = \frac{1}{\sqrt{2\epsilon}} u_p \exp^{-ipx}, \qquad (25)$$

**where** $u_p$ **is normalized bispinor, and** $\epsilon = +\sqrt{p^2 + m^2}$ **is the** ***positive frequency.***

**These spinor factors of plane wave, (25), being related with the solutions of the Kerr theorem** $Y^{\pm}(x)$**, acquire angular dependence (18) and (19), which corresponds to** ***spherical helicity states .***

**By using the correspondence of (18) with direction** $k^{\mu(+)}$**, we connect this congruence with** ***physical sheet*** **of the considered KN solution. However, setting consistency of the** ***left*** **spinor field** $\phi$ **with PNC of this sheet** $k^{\mu(+)(x)}$**, we obtain that the** ***right*** **spinor field** $\bar{\chi}$ **does not match with PNC** $k_{\mu(+)}(x)$**, but corresponds to antipodal null directions** $k^{\mu(-)}(x)$**, which belong to another sheet of the Kerr geometry.**

Therefore, the left-hand and right-hand fields of an electron, corresponding to antipodal directions of the Kerr congruence, cannot belong to the same sheet of the Kerr geometry. We obtain that massive solutions of the Dirac equation are incompatible with any one-sheet Kerr geometry and require presence of the second sheet of advanced fields. The left and right spinor fields should reside outside the soliton on the different sheets of the KN solution. The left spinor field $\phi_\alpha$ should propagate on the sheet, which is compatible with solution $Y^+(x)$ and the retarded congruence $k_\mu^{(+)}(x)$, while the right spinor field $\bar{\chi}^{\dot{\alpha}}$ should reside on the sheet corresponding to $Y^-(x)$ and has the advanced congruence $k_\mu^{(-)}(x)$.

Extending the left and right spinor fields inside the solitonic bag, we obtain that they transfer into the flat Minkowski space, where the both null congruences turn out to be compatible, so far as these congruences are null with respect to the same flat sheet of the common internal Minkowski space. Therefore, together with disappearance of the space-time ***curvature***, their incompatibility disappears too, and inside the soliton they are united into a Dirac bispinor $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$, corresponding to the massless Dirac equation.

$$(\gamma^\mu \partial_\mu)\Psi(x) = 0. \tag{26}$$

The localized inside the bag Higgs field $\mathcal{H}$ results in the Yukawa interaction

**between the left and right spinor fields**

$$\mathcal{L}_{Yukawa}(\mathcal{H}, \Psi) = -g\bar{\Psi}\mathcal{H}\Psi, \tag{27}$$

**which adds the mass term to the massless Dirac Lagrangian, generating the full Dirac equation with mass**

$$m \equiv g\mathcal{H}. \tag{28}$$

**Therefore, two antipodal solutions of the Kerr theorem** $Y^+(x)$ $Y^-(x) = -1/\bar{Y}^+(x)$, **corresponding to quadratic generating function** $F(T^A)$, **generate two massless spinor fields** $\phi_\alpha$ **and** $\chi^{\dot{\alpha}}$ **which form the massless bispinor Dirac field** $\Psi = (\phi_\alpha, \chi^{\dot{\alpha}})^T$. **Penetrating inside the bag, these fields meet the confined Higgs field** $\mathcal{H}$ **which connects them via Yukawa coupling (27) and turns them into massive Dirac field in the full correspondence with a basic role of the Higgs field in the Standard Model.**

Form of the solutions

***Compton region of the electron forms an oblate rotating vacuum bubble. The bubble is filled by coherently oscillating Higgs field ⇒ superconducting soliton.***

**Spinor solutions are localized in the narrow boundary of bubble – effectively similar to the SLAC-bag model.**

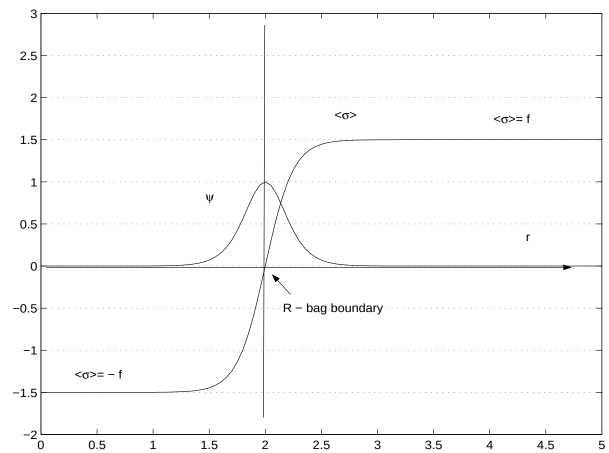


Figure 5: **Potential and the quark wave-function in the SLAC-bag model.**

**In the Dirac theory: *.on "...the distances comparable with $\hbar/mc$ ... the negative frequency amplitudes will then be appreciable, the zitterbewegung terms in the current important, and indeed we shall find ourselves beset by paradoxes and dilemmas which defy interpretation within the framework so far developed by the Dirac theory of an electron*" (J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics.)**
**COMPTON SIZE of the KN SOLITON BAG indicates its relation to a DRESSED electron, which has the SHAPE of a deformed false-vacuum bag:**
**RELATIVISTICALLY ROTATING OBLATE BUBBLE OF THE COMPTON RADIUS BOUNDED BY the CIRCULAR STRING with a quantized WILSON LOOP!**

***Vacuum bubble does not scatter, and the high-energy experiments cannot observe its structure, only the pointlike BARE ELECTRON is detected.***

**The KN point-like BARE ELECTRON is associated with singular end points (quarks) of the KN CIRCULAR STRING, similarly to point-like quarks of the SLAC-bag model.**
**String excitations (traveling waves) create "zitterbewegung", which acquires a natural explanation in the KN bag-soliton model.**

**CONCLUSION:**

- **the gravitating KN soliton model does not contradict to quantum theory,**
- **the KN soliton represents a false-vacuum bag with confined Higgs field,**
- **consistency of the KN gravity determines unambiguously shape of the bag as an oblate rotating disk,**
- **the bag of the KN soliton may be associated with a DRESSED electron,**
- **the false-vacuum state inside the bag is coherently oscillating with frequency** $\omega = 2m$.
- **a circular string is formed on the perimeter of the oblate bag, and circulating end points of the string (poles) correspond to zitterbewegung of the BARE electron.**

**THANK YOU FOR ATTENTION!**