Dark Energy to Modified Gravity

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The Big Puzzle

- 70% dark energy
- 25% dark matter
- 5% ordinary matter
Acceleration of the expansion

Dark Energy?

Modified gravity on large enough scales?
Acceleration of the expansion

Dark Energy?

\[ S = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} (R - \mathcal{L}_{DE}) \]
Acceleration of the expansion

Modified gravity on large enough scales?

Large scale structures
One may try to modify the effective equations of gravity on linear scales:

\[ ds^2 = a^2(\eta)(-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)dx^2) \]

Two Newtonian potentials related by:

\[ \Psi = \gamma(a, k)\Phi \]

And a modification of the Poisson equation and Newton’s law:

\[ -k^2\Phi = 4\pi G_N \nu(k, a) \bar{\rho}\delta \quad \theta' + \mathcal{H}\theta = k^2\Psi \]

Leading to a modification of the linear growth equation:

\[ \dddot{\delta} + \mathcal{H}\ddot{\delta} - \frac{3}{2}\gamma(a, k)\nu(a, k)\mathcal{H}^2\delta = 0 \quad \rightarrow \quad f_g(k, z) \]
EUCLID Forecast

Equation of state

\[ f_g(z) = \frac{d \ln \delta}{d \ln a} \]

Growth index

\[ f_g(z) = \Omega_m^\gamma \]

Model independent parameterisation valid on linear scales only.
Nothing guarantees that a modification of gravity on large scales is consistent with the gravity tests in the solar system.
Modified gravity on large enough scales.

Potential energy leads to dark energy.

Massive graviton always involves a scalar field.

Parameterised by a scalar field

\[ V(\phi) \]
For these ubiquitous scalars: very low masses
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Major gravitational problem!
Deviations from Newton’s law are parametrised by:

\[ \phi_N = -\frac{G_N}{r} (1 + 2\beta^2 e^{-r/\lambda}) \]

For large range forces with large \( \lambda \), the tightest constraint on the coupling \( \beta \) comes from the Cassini probe measuring the Shapiro effect (time delay):

\[ \beta^2 \leq 4 \cdot 10^{-5} \]
A large class of modified gravity models:

\[ S_{MG} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R) \]
f(R) is totally equivalent to an effective field theory with gravity and scalars!

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{Pl}}g_{\mu\nu}) \right) \]

The potential V is directly related to f(R)

\[ V(\phi) = m_{Pl}^2 \frac{R f' - f}{2f'^2}, \quad f' = e^{-2\phi/\sqrt{6}m_{Pl}} \]

Crucial coupling between matter and the scalar field
Are these models completely ruled out?

\( \beta = \frac{1}{\sqrt{6}} \)
SCREENING
Mechanisms whereby nearly massless scalars evade local gravitational tests
Around a background configuration and in the presence of matter, the Lagrangian can be linearised and the main screening mechanisms can be schematically distinguished:

\[
\mathcal{L} \supset -\frac{Z(\phi_0)}{2}(\partial \delta \phi)^2 - \frac{m^2(\phi_0)}{2}\delta \phi^2 + \frac{\beta(\phi_0)}{M_P}\delta \phi \delta T,
\]

The Vainshtein mechanism reduces the coupling by increasing Z.
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The Damour-Polyakov mechanism reduces the coupling \( \beta \).
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\]

The chameleon mechanism increases the mass.
The **Vainshtein** and **K-mouflage** mechanisms can be nicely understood:

Effective Newtonian potential:

\[ \Psi = (1 + \frac{2\beta^2(\phi)}{Z(\phi)})\Phi_N \]

For theories with second order eom:

\[ Z(\phi) = 1 + a(\phi)L^2\frac{D^\mu D_\mu \phi}{m_{Pl}} + b(\phi)\frac{(\partial \phi)^2}{M^4} + \ldots \]

\[ M^4 \sim 3H_0^2m_{Pl}^2, \quad L \sim H_0^{-1} \]
Newtonian gravity retrieved when the curvature is large enough:

\[ \nabla^2 \Phi_N \geq \frac{1}{2\beta L^2} \]

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened:

\[ \delta \geq \frac{1}{3\Omega_m^0 \beta} \]

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

\[ R_V = \left( \frac{3\beta m L^2}{4\pi m_p^2} \right)^{1/3} \]
K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

\[ |\nabla \Phi_N| \geq \frac{M^2}{2\beta m_{Pl}} \]

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened:

\[ \frac{k}{H_0} \leq \beta \delta \]

On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

\[ R_K = \left( \frac{\beta m}{4\pi m_{Pl} M^2} \right)^{1/2} \]

Dwarf galaxies are not screened.
The screening criterion for an object **BLUE** embedded in a larger region **RED** expresses the fact that the Newtonian potential of an object must be larger than the variation of the field:

Scalar charge:

\[ Q_A = \frac{|\phi_G - \phi_A|}{2m_{pl} \Phi_A} \]

\[ Q_A \leq \beta_G \]

**Self screening:** large Newton potential

**Blanket screening:** due to the environment \( G \)

\( \Phi_A \) Newton’s potential at the surface
The effect of the environment

When coupled to matter, scalar fields have a matter dependent effective potential

\[ V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1) \]

The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.
All these models can be entirely characterised by 2 time dependent functions. The non-linear potential and coupling of the model can be reconstructed using:

\[ \phi(a) = \phi_i + \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta(a) \rho(a)}{am^2(a)} \]

\[ V(a) = V_i - \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta^2(a) \rho^2(a)}{am^2(a)} \]

\[ \rho(a) = \frac{\rho_0}{a^3} \]
\[ f(R) = R - \Lambda_0 - \frac{f_{R0}}{n} \frac{R_0^{n+1}}{R^n} \]

\[ \beta = \frac{1}{\sqrt{6}}, \quad m_0 = H_0 \left( \frac{4\Omega_{\Lambda 0} + \Omega_{m0}}{(n + 1)|f_{R0}|} \right)^{1/2}, \quad m = m_0 \left( \frac{4\Omega_{\Lambda 0} + \Omega_{m0}a^{-3}}{4\Omega_{\Lambda 0} + \Omega_{m0}} \right)^{(n+2)/2} \]
The Milky Way must be screened

If not effects on the dynamics of satellite galaxies:

\[ \phi_C \leq 2\beta_C m_{Pl} \Phi_G \]

This gives a bound depending on the mass and coupling

\[ a_C = \begin{cases} 1 & \text{Self screening} \\ (200)^{-1/3} & \text{Blanket screening} \end{cases} \]

\[ 9\Omega_{m0} H_0^2 \int_{a_G}^{a_C} da \frac{\beta(a)}{a^4 m^2(a)} \leq 2\beta_C \Phi_G \]

\[ \Phi_G \sim 10^{-6} \quad a_G \sim 10^{-2} \]
Self-screening of the Milky Way:

\[ \frac{m_0^2}{H_0^2} \geq \frac{9 \Omega_{m0}}{2(2r-3-s)} 10^6 \]

This bounds the range of the scalar interaction to be less than a few Mpc’s on cosmological scales.
Due to the scalar interaction, within the Compton wavelength of the scalar field, the inertial and gravitational masses differ for screened objects:

\[ \eta_{BC} \equiv \left| \frac{a_C - a_B}{a_C + a_B} \right| = Q_A |Q_C - Q_B| \]

\[ \eta_{\text{moon-earth}} \approx Q_\oplus^2 \]

\[ \eta_{\text{moon-earth}} \leq 10^{-13} \]

\[ Q_\oplus \leq 10^{-7} \]
The Lunar Ranging constraint becomes:

\[
9 \Omega_{m0} H_0^2 \int_0^{a_G} da \frac{\beta(a)}{a^4 m^2(a)} \leq 10^{-7} \Phi_+ \quad \Phi_+ \sim 10^{-9}
\]

This leads to a tight bound on the range:

\[
\frac{m_0^2}{H_0^2} \geq 9 \Omega_{m0} \beta_0 \frac{3n+3}{a_{G}^3} a_{G}^{3n+3} 10^{16}
\]

Large curvature f(R): for n>1 the Milky Way condition is the strongest.

\[
|f_{R_0}| \leq 10^{-6}
\]
Big Bang Nucleosynthesis tells us that particle masses should not vary more than 10% between BBN and now.

This is realised provided that:

The field follows the minimum of the effective potential since BBN.

The mass is always much larger than the Hubble rate $m >> H$

The equation of states varies very little from the concordance model:

$$1 + w = \mathcal{O}(\frac{H^2}{m^2}) \leq 10^{-6}$$

At the background level, these models are indistinguishable from $\Lambda$-CDM.
At the linear level, CDM perturbations grow differently from GR:

\[
\delta''_c + \mathcal{H} \delta'_c - \frac{3}{2} \Omega_m \mathcal{H}^2 (1 + \frac{2\beta(a)^2}{1 + \frac{m(a)^2 a^2}{k^2}}) \delta_c = 0
\]

Inside the Compton wavelength \( k \ll m(a)a \), anomalous growth depending on the coupling to matter \( \beta(a) \).

\[
\lambda_c = \frac{1}{ma}
\]

Outside the Compton wavelength, growth is not modified:

\( \delta \sim a \)

Inside the Compton wavelength, more growth:

\[
\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}
\]
\[ \frac{m_0}{H_0} \geq 10^3 \]

Modification of gravity on quasi-linear to non-linear scales

N-body simulations:

ECOSMOG simulations using a modification of the RAMSES code.
Summary

Dark Energy/Modified gravity requires low mass fields, and therefore fifth force problems

Cured by screening mechanisms: Chameleon, Damour-Polyakov or Vainshtein

Strong constraints on the interaction range leading to implication on quasi-linear structures of the Universe