The cosmological constant and quantum vacuum.

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astrophysique & planétologie

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Université de Toulouse

IUF

Accelerated expansion

There is no FL model that reproduces the present day observations without acceleration...

Nobel Prize in Physics 2011

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S.Perlmuter, A.Riess, B.Schmidt

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Observations need $P \approx -\rho$
So that the gravity strength is repulsive and proportional to R

...

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So is this the origin of the acceleration ?

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The Vacuum catastroph (Weinberg, 1989):

$$\rho_v = \langle 0 | T^{00} | 0 \rangle = \frac{1}{(2\pi)^3} \int_0^{+\infty} \frac{1}{2} \hbar\omega \, \mathrm{d}^3\mathbf{k}$$

with $\omega^2 = k^2 + m^2$

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with $\omega^2 = k^2 + m^2$ highly divergent:

$$\rho_v(k_c) \propto \frac{k_c^4}{16\pi^2}$$

(for $k_c \gg m$).

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1/3}) \sum_i \langle 0 | T^{ii} | 0 \rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

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$\rightarrow$ usual conclusion on zero-point energy contribution (for instance by dimensional regularization).

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cf Review by J.Martin 2012 (astro-ph/1205.3365).

Everything You Always Wanted To Know About
The Cosmological Constant Problem (But Were Afraid To Ask)

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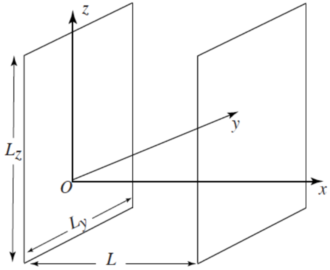

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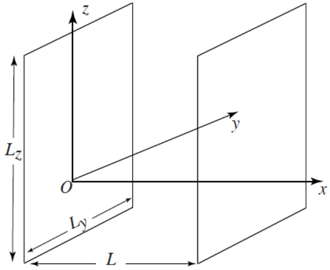

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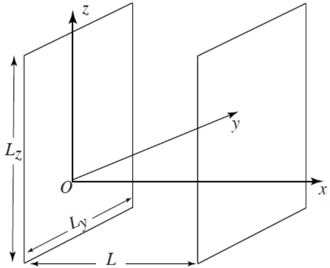

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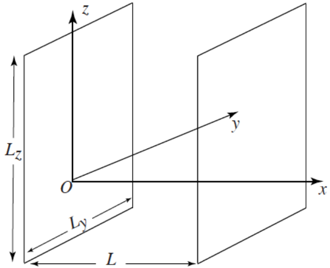

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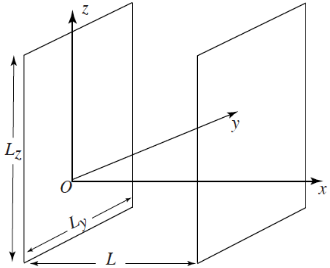

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Brown & Maclay (1968)

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This (permanent) contribution can be evaluated by mean of dimensional regularization.
But : $\rho < 0$

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Assumption 1: At high energy, only modes with λ smaller than ct have to be taken into account i.e.:

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{\omega>\omega_H}^{\infty} k^2 \mathrm{d}k \left[\sum_{n=-\infty}^{\infty} \left(k^2 + \frac{n^2}{R^2} \right)^{1/2} \right]$$

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Assumption 2: as long as $ct \ll \pi R$ gravitational vacuum should be that of a massless field in a 4+1D space time i.e.:

$$\rho_v = 0$$

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Later, when $ct \gg \pi R$ i.e. $\omega_H \sim 0$

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{0}^{\infty} k^2 \mathrm{d}k \, [...] = \frac{5\hbar c}{8\pi^3 R} \int_{0}^{1/R} k^2 \mathrm{d}k \, [...]$$

with :

$$[...] = \left[\sum_{n=-\infty}^{\infty} \left(k^2 + \frac{n^2}{R^2} \right)^{1/2} \right]$$

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The condition :

$$\omega = \sqrt{k^2 + \frac{n^2}{R^2}} < \frac{1}{R}$$

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In the brane:

$$\rho_v = \frac{5\hbar c}{16\pi^2 R^4}$$

Dark energy emerges...

Pressure:

$$P_v^{\perp} = 4\rho_0 = \frac{20\hbar c}{32\pi^3 R^5}$$

Along the brane, using the fact that the $T^{\mu\nu}$ is traceless and integrating along the 4th spatial dimension:

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$\Omega_v \sim 0.7 \Rightarrow R \sim 35\mu\mathrm{m}$ fits data. Corresponding to $E \sim 1\,TeV$

Consequences

Acceleration is due to vacuum: GR + $w = -1$

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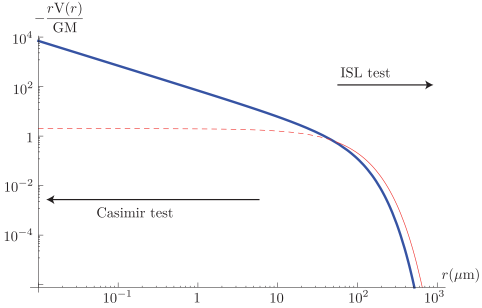
The presence of additional compact "large" dimension ($\sim 35\mu\mathrm{m}$) can be tested by experiment on gravitational inverse square law on short scale. Additional term:



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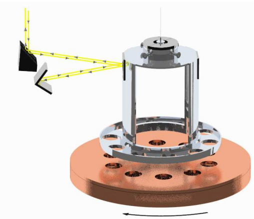

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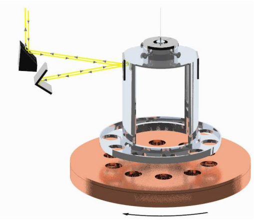
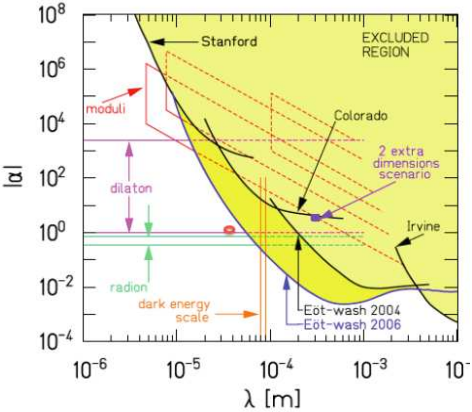


Present day limit (Kapner et al. 2007; Adelberger et al. 2009) :

Testing the idea



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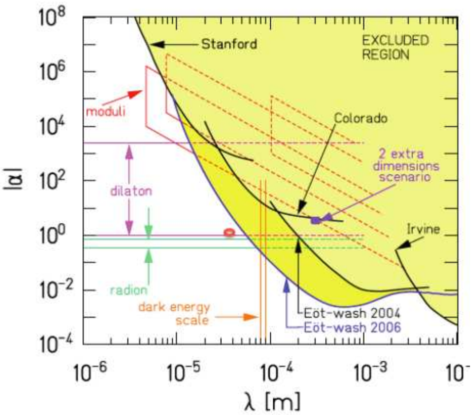

$$R < 46\mu\mathrm{m}$$

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- $\rightarrow$ Testable!