Analytical continuation of Black Hole entropy in Loop Quantum Gravity

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Introduction

Purposes

- Defining a consistent analytical continuation of the entropy for black hole with the complex Ashtekar variables

Context

- Black hole in Loop Quantum gravity : treated as boundary in the spacetime (isolated horizon)
- Gravitational D.o.F on the horizon described by a Chern Simons theory living on this horizon with the constraint :

$$k = \frac{(1-\gamma^2)A_h}{2\pi\gamma}$$

- For real Black Hole, γ plays a crucial role :

$$S_{bh} = \frac{A_h \gamma_0}{4\gamma} \qquad \gamma = \gamma_0 \sim 0,2375$$

Ideas

Problematic

- What happen if one want to work with $\gamma = i$? [Frodden, Geiller, Noui, Perez (2012)]

Motivations

- γ plays no role at the classical level
- Ashtekar-Barbero not a full space-time connection [Samuel, Alexandrov]
- Three dimensional gravity : toy model [Ben Achour, Geiller, Noui, Yu (2013)]
  - to solve the dynamic, need to work with an su(1,1)-connection
  - area spectrum becomes continuous and independent of γ

$$A(s) = 8\pi l_p^2 \sqrt{s^2 + 1/4} \qquad s \in \mathbb{R}^+$$

Analytical continuation : first step

Problematic

- What happen if one want to work with $\gamma = i$? [Frodden, Geiller, Noui, Perez (2012)]

The Chern Simons level become purely imaginary

- To keep an area real and positive, one need to send $k \to i\lambda$ when $\gamma = i$.

$$A_h = \frac{2\pi\gamma k}{1-\gamma^2} \quad \gamma \in \mathbb{R} \qquad \to \qquad A_h = 2\pi\lambda \quad \gamma = i$$

- Problem : defining an analytical continuation of the Chern-Simons theory for $k \to i\lambda$ [Witten (2010), Morse theory]

Analytical continuation : first step

Rewriting the Verlinde formula as an integral in the complex plane

- Dimension of the Chern Simons Hilbert space $SU_q(2)$:

$$N_k(d_l) = \frac{2}{k+2} \sum_{d=1}^{k+1} sin^2(\frac{\pi d}{k+2}) \prod_{l=1}^{n} \frac{sin(\frac{\pi}{k+2} d d_l)}{sin(\frac{\pi}{k+2} d)} \quad d_l = 2j_l + 1$$

- Can be reinterpreted as a sum of residues of the following integral :

$$I_k(d_l) = \frac{i}{\pi} \oint_{\mathcal{C}} sinh^2 z \prod_{l=1}^{n} \frac{sinh(d d_l)}{sinh(z)} coth(k+2)z \quad (k, d_l) \in \mathbb{N}$$

- Poles in $z_p = \frac{i\pi p}{k+2}$ with $p \in \mathbb{N}^*$
- Contour $\mathcal{C}$ encloses the imaginary axis between $[0, i\pi]$

Analytical continuation : second step

Starting point

- Case $(d_l, k) \in \mathbb{N}$: pole in $z = \frac{i\pi p}{k+2} \quad p \in \mathbb{N}$, on the imaginary axis

Naive analytical continuation : $k \to i\lambda$

- Case $d_l \in \mathbb{N} \quad k \in i\,\mathbb{R}$: pole in $z = -\frac{\pi p}{\lambda} \quad p \in \mathbb{N}^*$, on the real axis
- Integral vanish since the contour doesn't enclose any pole : inconsistent

Consistent analytical continuation : $k \to i\lambda$, $d \to i\,s$

- Case $d_l \in i\,\mathbb{R} \quad k \in i\,\mathbb{R}$: pole in $z = -\frac{\pi p}{\lambda} \quad p \in \mathbb{N}$, on the real axis and pole in $z = i\,\pi m \quad m \in \mathbb{N}^*$ on the imaginary axis
- Only way to have a nontrivial analytical continuation, make d purely imaginary : $d = 2j + 1 \to d = i\,s$, $j \to -\frac{1}{2} + is$, $s \in \mathbb{R}^+$

Thermodynamical study

Definition of the number of microstates

- $\gamma = i,\ k = i\lambda,\ d = is$
  For large black hole , $\lambda \gg 1$

$$I_\infty(s,n) = \frac{i}{\pi} \oint_{\mathcal{C}} sinh^2 z \prod_{l=1}^{n} \frac{sinh(sz)}{sinh(z)} \quad s \in \mathbb{R}$$

One color model

- Same color for each puncture : one color model

$$I_\infty(s,n) = \frac{i}{\pi} \oint_{\mathcal{C}} sinh^2(z)\ e^{nS(z)} \quad S(z) = log \frac{sinh(sz)}{sinh(z)}$$

- The thermodynamical limit : $n \gg 1$, stationnary phase method

Partition function and entropy

Microcanonical ensemble

- Entropy (with a Gibbs factor)

$$S = \frac{a_H}{4 l_p^2} + B\sqrt{\frac{a_H}{2\pi l_p^2}} - 2log(\frac{a_H}{l_p^2})$$

Grand canonical ensemble

- Partition function
  Indistinguishibilty of the punctures [Gosh, Noui, Perez (2014)]

$$Z = \int ds \sum \frac{1}{n!} g(n,s) e^{-\beta E}$$

- Frodden-Gosh-Perez notion of energy : $E = \frac{A}{8\pi L}$.

$$Z(\beta) \sim \sqrt{8\pi} x^3 exp(\frac{1}{2x}) \qquad x = \frac{l_p^2}{2L}(\beta - \beta_U)$$

Grand canonical ensemble

Entropy with $\mu = 0$

- Mean area

$$< a_H > = \frac{2\pi l_p^2}{x^2}$$

- Thermodynamical limit : $x \to 0$.
- Mean number of puncture and mean color

$$< n > = \frac{1}{8\pi}\sqrt{< a_H >} \qquad < s > = \frac{1}{4\pi}\sqrt{< a_H >}$$

- Semiclassical regim dominated by large spins !
- Entropy

$$S = \frac{a_H}{4 l_p^2} + \sqrt{\frac{a_H}{2\pi l_p^2}} - \frac{3}{2} log(\frac{a_H}{l_p^2})$$

Grand canonical system

Entropy when $\mu = 2T_U$

- Non zero chemical potential for the punctures : $\mu = 2T_U$
- Entropy :

$$S = \frac{a_H}{4l_p^2} - \frac{3}{2} log(\frac{a_H}{l_p^2})$$

- Non vanishing chemical potential / black hole release energy when one remove a puncture = radiation

Main result

- With $\gamma = i$, the Bekenstein Hawking area law is recovered without any unnatural fine tunning + logarithmic corrections

Conclusion and perspectives

General perspectives for black hole physics

- Define a unique and consistent analytical continuation of black hole entropy
- Introducing quantum statistic for the punctures gas

General perspectives for LQG

- Give strong indications how to work on the self dual side of the theory
  Precise prescription for the analytical continuation
- Applying the same prescription to the real LQC [Ben Achour, Grain, Noui (2014) arXiv :1407.3768 [gr-qc]]
- Could give new ideas to resolve the reality conditions, [Thiemann, Ashtekar (1995)]
- Give new insight on the status of the Barbero Immirizi parameter : regulator to be send to *i* to get physical predictions