

Frontiers of Fundamental Physics 14

List of speakers in conference

Quantum Gravity

Updated on January 8, 2015

Tim **Adamo** (DAMTP)

July, 15, 14h30 – 15h00, Room 404, Quantum Gravity

Gravitational Scattering via Twistor Theory

Scattering amplitudes are among the most natural—and important—observables of any field theory in an asymptotically flat space-time, including any theory of quantum gravity. Recently, gravitational scattering amplitudes have been shown to possess remarkably compact expressions which are un-expected from the perspective of traditional Lagrangian-based perturbation theory (c.f., [1]). We will discuss how some of these formulae arise from working with an action principal not on space-time, but rather in *twistor space*, an auxiliary complex three-manifold [2]. This narrative exploits an on-shell equivalence between conformal gravity and general relativity in asymptotically de Sitter manifolds [3], and also leads to new expressions for analogues of scattering amplitudes in space-times with a positive cosmological constant.

References

- [1] F. Cachazo & D. Skinner, *Gravity from Rational Curves in Twistor Space*, *Phys.Rev.Lett.*, **110** (2013) 161301 arXiv:1207.0741.
- [2] T. Adamo & L. Mason, *Conformal and Einstein gravity from twistor actions*, *Class.Quant.Grav.*, **31** (2014) 045014 arXiv:1307.5043.
- [3] J. Maldacena, *Einstein gravity from conformal gravity*, (2011) arXiv:1105.5632.

Emanuele **Alesci** (UW)

July, 18, 15h30 – 16h00, Room 404, Quantum Gravity

Quantum Reduced Loop Gravity

We will review the current developments of Quantum Reduced Loop Gravity [1,2], a recently proposed model to address the quantum dynamics of the early Universe. In particular we will discuss its semiclassical limit, a link with LQC [3] and will review how the QRLG technique naturally selects states based on coherent intertwiners that could simplify the analysis of the dynamics in the full theory [4].

References

- [1] E. Alesci and F. Cianfrani, *A new perspective on cosmology in Loop Quantum Gravity*, *Europhys. Lett.* **104**, 10001 (2013); arXiv:1210.4504 [gr-qc].
- [2] E. Alesci and F. Cianfrani, *Quantum-Reduced Loop Gravity: Cosmology*, *Phys. Rev. D* **87**, no. 8, 083521 (2013); arXiv:1301.2245 [gr-qc].
- [3] E. Alesci and F. Cianfrani, *Quantum Reduced Loop Gravity: Semiclassical limit*, arXiv:1402.3155 [gr-qc].
- [4] E. Alesci, F. Cianfrani and C. Rovelli, *Quantum-Reduced Loop-Gravity: Relation with the Full Theory*, *Phys. Rev. D* **88**, 104001 (2013); arXiv:1309.6304 [gr-qc].

Sergey **Alexandrov** (UM2)

July, 15, 15h00 – 15h30, Room 404, Quantum Gravity

First order gravity on the light front

After a brief review of unusual features of the light front canonical formulations of field theories, I present the analysis of the canonical structure of the first order formulation of general relativity on a lightlike foliation. It appears to be quite different from the usual spacelike case leading, for instance, to the presence of tertiary constraints. Besides, I discuss the issue of the zero modes and argue that there might be some hidden correspondence with two-dimensional theories.

How many quanta are there in a quantum spacetime?

We develop a technique for describing quantum states of the gravitational field in terms of coarse grained spin networks, following earlier insights by Livine and Terno [1], [2]. This technique shows that the number of nodes and links in a quantum state of gravity, and the spin associated to the links, depend on the observables chosen for the description of the state, and therefore the question of the title is ill posed, unless further information about what is been precisely measured is given.

References

- [1] S. Ariwahjoedi, J. S. Kosasih, C. Rovelli, F. P. Zen, *How many quanta are there in a quantum spacetime?* [gr-qc/1404.1750]
- [2] E. R. Livine and D. R. Terno, *Bulk entropy in loop quantum gravity*, Nuclear Physics B 794 (May, 2008) 138-153, [gr-qc/0706.0985].
- [3] E. R. Livine and D. R. Terno, *Reconstructing Quantum Geometry from Quantum Information: Area Renormalisation, Coarse-Graining and Entanglement on Spin Networks*, [gr-qc/0603008].
- [4] D. Colosi and C. Rovelli, *What is a particle?*, Class. Quant. Grav. 26 (2009) 25002, [gr-qc/0409054].

Application of the Curvature operator: Matrix Elements and properties of the new Hamiltonian Constraint operator in LQG

This talk is a presentation of an ongoing work based on [1] of the same authors. In this work we study properties of the Lorentzian Hamiltonian constraint operator expressed using the curvature operator introduced in [1] and we evaluate its action.

References

- [1] E. Alesci, M. Assanioussi and J. Lewandowski, *A curvature operator for LQG*, Submitted for publication in Phys. Rev. D [arxiv: 1403.3190v2].

On background-independent renormalization in spin foam models

In recent years, spin foam models have been proposed to define a path integral for quantum gravity in a non-perturbative and background-independent way.

In this talk I will address the questions of continuum limit and diffeomorphism symmetry in these models, and relate them to the notion of Wilsonian renormalization group flow. A proposal for a background-independent formulation of renormalization for spin foam models will be presented, which does not require a notion of length scale. The concept of cylindrical consistency plays a crucial role in this context, and we show how it naturally defines the RG equations, providing a continuum limit for the theory.

Analytical continuation of black hole entropy in Loop Quantum Gravity

Recovering the Bekenstein-Hawking formula for black hole entropy is a crucial test for any candidate to a quantum theory of gravity. The approach followed in Loop Quantum Gravity leads successfully to the famous formula, up to a choice for the real Barbero-Immirzi parameter, plus logarithmic corrections in agreement with other approaches. However, it has been recently shown in [1] that proceeding to an analytic continuation of the dimension of the Hilbert space of the Chern Simons theory (related to the degrees of freedom of the horizon) leads directly to the exact Bekenstein-Hawking formula (without recovering the logarithmic corrections). Therefore, working with the self dual Ashtekar connection seems to give directly the right result. Such a result underlies the status of the complex Ashtekar variables as the good connection to use in Loop Quantum Gravity, additional works pointing in the same direction [2][3][4][5][6]. Yet, a rigorous construction of this analytical continuation was still missing. This is precisely what was done in a recent article and this is the subject of this talk. I will first focus on the construction of the analytical continuation. Then, I will detail the main result, that is the Bekenstein-Hawking formula and its logarithmic corrections in the context of Self Dual Loop Quantum Gravity.

References

- [1] E. Frodden, M. Geiller, K. Noui and A. Perez, *Black hole entropy from complex Ashtekar variables*, (2012), arXiv:1212.4060 [gr-qc].
- [2] J. Samuel, *Is Barbero's Hamiltonian formulation a gauge theory of Lorentzian gravity?*, Class. Quant. Grav. 17 L141 (2000), arXiv:gr-qc/0005095.
- [3] S. Alexandrov, *On choice of connection in loop quantum gravity*, Phys. Rev. D 65 024011 (2002), arXiv:gr-qc/0107071.

Coherent State Operators in Cosmology and Gravity

Coherent States (CS) are widely used in physics, and quantum gravity is not an exception. However, the application of CS in the construction of operators is rather unexplored in the areas of quantum gravity and quantum cosmology. In my talk, I will present how CS can be used to define “coherent state operators” via a procedure known as “coherent state quantization” [1]. This procedure produces operators with inbuilt good semiclassical properties, while at the same time preserving typical quantum attributes (e.g., discreteness of spectra). Examples of such coherent state quantization will cover: (1) operators on homogeneous isotropic quantum cosmology [2] (based on the affine group); (2) possible extensions to Bianchi I case; (3) simple operators on $L_2(SU(2), d\mu_H)$.

References

- [1] S. Twareque Ali, J.-P. Antoine and J.-P. Gazeau, *Coherent States, Wavelets, and Their Generalizations*, Springer-Verlag (2013)
 [2] H. Bergeron, A. Dapor, J.-P. Gazeau and P. Malkiewicz, *Smooth big bounce from affine quantization*, Phys. Rev. D **89**, 083522 (2014)

A brief overview of loop quantum cosmology and its potential observational signatures

Loop quantum cosmology (a symmetry-reduced quantum model of the Universe inspired by loop quantum gravity) extends the inflationary paradigm to the Planck era: the big bang singularity is replaced by a quantum bounce naturally followed by inflation. Testing for these models requires to compute the amount of cosmological perturbations produced in this quantum background and subsequently derives their footprints on the cosmic microwave background. I propose to review two theoretical approaches treating for cosmological perturbations in a quantum background [1,2], making their respective assumptions and methodology as explicit as possible. I will then show the observational consequences of those treatments focusing on the specific case of the cosmic microwave background anisotropies as a probe of the primordial Universe [3].

References

- [1] A. Barrau, M. Bojowald, G. Calcagni, J. Grain & M. Kagan, *Anomaly-free cosmological perturbations in effective canonical quantum gravity*, arXiv:1404.1018.
 [2] I. Agullo, A. Ashtekar & W. Nelson, *Extension of the quantum theory of cosmological perturbations to the Planck era*, Phys. Rev. D **87** (2013) 043507
 [3] A. Barrau, T. Cailleteau, J. Grain & J. Mielczarek, *Observational issues in loop quantum cosmology*, Class. Quant. Grav. **31** (2014) 053001.

Curved polyhedra

We describe how to reconstruct a constant curvature tetrahedron given four holonomies whose product is the identity. We construct a phase space describing the shapes of curved tetrahedra. We give a conjecture for the generalization of Minkowski’s theorem to spaces of constant curvature. This establishes a new route to the description of four dimensional quantum gravity with a cosmological constant.

Matter Bounce Scenario in $F(T)$ gravity

The CMB map provided by the *Planck* project constrains the value of the ratio of tensor-to-scalar perturbations, namely r , to be smaller than 0.11 (95 % CL). This bound rules out the simplest models of inflation. However, recent data from BICEP2 is in strong tension with this constrain, as it finds a value $r = 0.20^{+0.07}_{-0.05}$ with $r = 0$ disfavored at 7.0σ , which allows these simplest inflationary models to survive. The remarkable fact is that, even though the BICEP2 experiment was conceived to search for evidence of inflation, its experimental data matches correctly theoretical results coming from the matter bounce scenario (the alternative model to the inflationary paradigm). More precisely, most bouncing cosmologies do not pass *Planck*’s constrains due to the smallness of the value of the tensor/scalar ratio $r \leq 0.11$, but with new BICEP2 data some of them fit well with experimental data. This is the case with the matter bounce scenario in $F(T)$ gravity.

Quantum formalism for systems with temporally varying discretization

A canonical quantum formalism for discrete systems subject to a discretization changing dynamics is outlined. This framework enables one to systematically study (non-)unitarity of such dynamics, the role of canonical constraints and the fate of Dirac observables on temporally varying discretizations. It will be illustrated how the formalism can also be employed to generate a vacuum for a scalar field on an evolving lattice. Implications for the dynamics in simplicial quantum gravity models are commented on.

References

- [1] P. A. Höhn, “Quantization of systems with temporally varying discretization I: Evolving Hilbert spaces” arXiv:1401.6062 [gr-qc].
- [2] P. A. Höhn, “Quantization of systems with temporally varying discretization II: Local evolution moves” arXiv:1401.7731 [gr-qc].
- [3] B. Dittrich and P. A. Höhn, “Constraint analysis for variational discrete systems” J.Math. Phys.54, 093505 (2013) [arXiv:1303.4294 [math-ph]].
- [4] B. Dittrich, P. A. Höhn and T. Jacobson, *to appear*

Exact formulation of the quantum scalar constraint in LQG

Several new applications for LQG will be presented. The first one is a new quantum representation of the gravitational scalar constraint. In this representation, for the first time in the literature, the quantum $C(N)$ itself preserves the Hilbert space for every lapse function N . Owing to that property, solutions to the quantum constraints of vacuum GR can be well defined by the spectral decomposition and set a physical Hilbert space. Our representation admits also a new proposal of the quantum physical Hamiltonian of the Rovelli-Smolin model of gravity coupled to the mass-less Klein-Gordon time field. The third application we propose, is a new operator for the quantum Hamiltonian of the Klein-Gordon Scalar field coupled to LQG. Our new framework captures the degrees of freedom of the scalar field lost in the framework in which time is deparametrized by the scalar field.

LQC on curved FLRW space time

This talk is based on [1,2]. Loop quantum cosmology predicts a bounce instead of the big bang. Anisotropies grow in a contracting universe and should therefore not be neglected at the bounce. In this talk I will discuss how the bounce and the following inflation is affected by anisotropies in effective Bianchi-I loop quantum cosmology.

References

- [1] L. Linsefors and A. Barrau, *Modified Friedmann equation and survey of solutions in effective Bianchi-I loop quantum cosmology*, Class.Quant.Grav. **31** (2014) 015018 [arXiv:1305.4516 [gr-qc]].
- [2] L. Linsefors and A. Barrau, *Duration of inflation in effective Bianchi-I loop quantum cosmology*, In preparation

Do interacting ultraviolet fixed point exist, and if so, what can we do with them?

It is widely acknowledged that the high-energy behaviour of quantum field theories should be governed by an ultraviolet fixed point. The fascinating idea that such a fixed point could be interacting, known as asymptotic safety, suggests that metric quantum gravity may well be fundamental in its own right

- [1] D. F. Litim, *Renormalisation group and the Planck scale*, Phil.Trans Roy.Soc. Lond. A **369** (2011) 2759 [arXiv:1102.4624 [hep-th]].
- [2] D. F. Litim and F. Sannino, *Asymptotic safety guaranteed*, arXiv:1406.2337 [hep-th].
- [3] D. F. Litim, *Fixed points of quantum gravity*, Phys. Rev.Lett. **92** (2004) 201301 [hep-th/0312114].
- [4] K. Falls, D. F. Litim, K. Nikolakopoulos and C. Rahmede, *A bootstrap towards asymptotic safety*, arXiv:1301.4191 [hep-th].
- [5] K. Falls, D. F. Litim and A. Raghuraman, *Black Holes and Asymptotically Safe Gravity*, Int. J.Mod. Phys. A **27** (2012) 1250019 [arXiv:1002.0260 [hep-th]]. [6] K. Falls and D. F. Litim, *Black hole thermodynamics under the microscope*, Phys. Rev. D **89** (2014) 084002 [arXiv:1212.1821 [gr-qc]].
- [7] D. F. Litim and K. Nikolakopoulos, *Quantum gravity effects in Myers-Perry space-times*, JHEP **1404** (2014) 021 [arXiv:1308.5630 [hep-th]].

Spinorial Path Integral for Loop Gravity: Coherent states and Spinfoam symmetries

The spinfoam framework defines transition amplitudes for spin network states of loop quantum gravity. I will review their recent reformulation in terms of spinorial variables allowing to see these amplitudes as coherent state path integrals [1,2,3]. This clarifies their geometrical meaning and at the identification of symmetries: recursion relations for $3nj$ symbols are turned into Hamiltonian constraints satisfied by the spinfoam amplitudes [4,5]. This applies in particular to the derivation of modified FRW equations for quantum cosmology [6].

References

- [1] L Freidel and S Speziale, *From twistors to twisted geometries*, Phys. Rev. D **82**, 084041 (2010) arXiv:1006.0199 [gr-qc]
- [2] E F Borja, L Freidel, I Garay and E R Livine, *$U(N)$ tools for Loop Quantum Gravity: The Return of the Spinor*, Class. Quant. Grav. **28**, 055005 (2011) arXiv:1010.5451 [gr-qc]
- [3] M Dupuis and E R Livine, *Holomorphic Simplicity Constraints for 4d Riemannian Spinfoam Models*, J. Phys. Conf. Ser. **360**, 012046 (2012) arXiv:1111.1125 [gr-qc]
- [4] V Bonzom, E R Livine and S Speziale, *Recurrence relations for spin foam vertices*, Class. Quant. Grav. **27**, 125002 (2010) arXiv:0911.2204 [gr-qc]
- [5] V Bonzom and E R Livine, *Generating Functions for Coherent Intertwiners*, Class. Quant. Grav. **30**, 055018 (2013) arXiv:1205.5677 [gr-qc]
- [6] E R Livine and M Martin-Benito, *Classical Setting and Effective Dynamics for Spinfoam Cosmology*, Class. Quant. Grav. **30**, 035006 (2013) arXiv:1111.2867 [gr-qc]

Quantum Gravity and the Foundations of Quantum Theory

Starting from the guiding principles of spacetime locality and operationalism, we consider a general framework for a probabilistic description of nature. Crucially, no notion of time or metric is assumed, neither any specific physical model. Remarkably, the emerging framework converges with a recently proposed formulation of quantum theory [1], obtained constructively from known quantum physics.

References

- [1] R. Oeckl, *A positive formalism for quantum theory in the general boundary formulation*, Found. Phys. **43** (2013) 1206–1232 [arXiv:1212.5571].

A quantum field theory for the atoms of space

We give a brief introduction to the group field theory (GFT) formalism for quantum gravity, a 2nd quantised reformulation of loop quantum gravity and spin foam models and a group-theoretic enrichment of the purely combinatorial tensor models. We then review some recent key developments concerning: the definition and analysis of 4d gravity models, progress in GFT renormalisation, and the extraction of effective cosmological dynamics from GFT condensates.

Black hole entropy and entanglement of Planckian degrees of freedom

We analyze the relationship between entanglement (or geometric) entropy with statistical mechanical entropy of horizon degrees of freedom when described in the framework of isolated horizons in loop quantum gravity. We show that, once the relevant degrees of freedom are identified, the two notions coincide. The key ingredient linking the two notions is the structure of quantum geometry at Planck scale implied by loop quantum gravity, where correlations between the inside and outside of the black hole are mediated by eigenstates of the horizon area operator.

Black holes in Asymptotically Safe Gravity

Black holes are probably among the most fascinating objects populating our universe. Their characteristic features, encompassing spacetime singularities, event horizons, and black hole thermodynamics, provide a rich testing ground for quantum gravity ideas. In this talk, we review the status of black holes within Weinberg's asymptotic safety program [1].

The resulting quantum improved Schwarzschild black hole is discussed in detail and the effective geometry will be compared to recent findings within Loop Quantum Gravity [2]. Moreover the quantum singularity emerging for the inclusion of a cosmological constant [3] is elucidated and linked to the phenomenon of a dynamical dimensional reduction of spacetime.

References

- [1] B. Koch and F. Saueressig, *Black holes within Asymptotic Safety*, Int J Mod Phys A **29** (2014) 8, 1430011 arXiv:1401.4452.
- [2] C. Rovelli and F. Vidotto, arXiv:1401.6562.
- [3] B. Koch and F. Saueressig, *Structural aspects of asymptotically safe black holes*, Class Quant Grav **31** (2014) 015006, arXiv:1306.1546.

Tatyana **Shestakova** (SFEDU)

July, 17, 18h00 – 18h30, Room 404, Quantum Gravity

The role of BRST charge as a generator of gauge transformations in quantization of gauge theories and Gravity

In the Batalin-Fradkin-Vilkovisky (BFV) approach to quantization of gauge theories a principal role is given to the BRST charge which can be constructed as a series in Grassmannian (ghost) variables with coefficients given by generalized structure functions of constraints algebra. Alternatively, the BRST charge can be derived making use of the Noether theorem and global BRST invariance of the effective action. In the case of Yang - Mills fields the both methods lead to the same expression for the BRST charge, but it is not valid in the case of General Relativity. It is illustrated by examples of an isotropic cosmological model as well as by spherically-symmetric gravitational model which imitates the full theory of gravity much better. The consideration is based on Hamiltonian formulation of General Relativity in extended phase space (1, 2). At the quantum level the structure of the BRST charge is of great importance since BRST invariant quantum states are believed to be physical states. Thus, the definition of the BRST charge at the classical level is inseparably related to our attempts to find a true way to quantize Gravity.

References

- [1] T. P. Shestakova, *Class. Quantum Grav.* **28** (2011), 055009.
- [2] T. P. Shestakova, to be published in: *Grav. Cosmol.* **20** (2014) p. 67-79.

Jędrzej **Swieżewski** (FUW, UW)

July, 18, 17h30 – 18h00, Room 404, Quantum Gravity

Observers diffeomorphism-invariant description of a general relativistic system

I will present a construction of observables following naturally from an introduction of an observer into a relativistic theory. The observables are invariant under a large class of spatial diffeomorphisms. I will discuss the description of the theory in terms of spatial-diffeomorphism-invariant degrees of freedom. The talk will be based on [1].

References

- [1] P. Duch, W. Kaminski, J. Lewandowski and J. Swieżewski, *Observables for General Relativity related to geometry*, arXiv:1403.8062 [gr-qc].

Johannes **Thürigen** (AEI)

July, 17, 15h00 – 15h30, Room 404, Quantum Gravity

Group field theories generating polyhedral complexes

A criticism on group field theories (GFT) as candidate definitions of the dynamics of Loop quantum gravity (LQG) has been the restriction of its boundary states defined on graphs to fixed valency. In this contribution we will review the classes of spacetime complexes common in LQG and give a definitions in terms of combinatorial complexes. We will then present a class of GFTs generating all of these, including the so called KKL spin-foam models.

Reiko **Toriumi** (CPT Marseille)

July, 17, 17h30 – 18h00, Room 404, Quantum Gravity

Cosmological constant: its identification as renormalization group invariant scale corresponding to a gravitational condensate

We examine the general issue of whether a scale dependent cosmological constant can be consistent with general covariance, a problem that arises naturally in the treatment of quantum gravitation where coupling constants generally run as a consequence of renormalization group effects. The issue is approached from several points of view, which include the manifestly covariant functional integral formulation, covariant continuum perturbation theory about two dimensions, the lattice formulation of gravity, and the nonlocal effective action and effective field equation methods. In all cases we find that the cosmological constant cannot run with scale, unless general covariance is explicitly broken by the regularization procedure.

References

- [1] H. W. Hamber and R. Toriumi, "Inconsistencies from a Running Cosmological Constant", *Int. J. Mod. Phys. D* **22**, no. 13, 1330023 (2013) [arXiv:1301.6259 [hep-th]].

Wolfgang **Wieland** (IGC)

July, 15, 16h00 – 16h30, Room 404, Quantum Gravity

Loop gravity from a spinorial action

Spinors have a wide range of applications, from quantum mechanics to particle physics, quantum information and general relativity [1]. In this talk, I will argue that they are useful also for discretized gravity, and present a version of first-order Regge calculus with spinors as the fundamental configuration variables [2]. The underlying action describes a mechanical system with finitely many degrees of freedom, the system has a Hamiltonian, and local gauge symmetries. I will derive the resulting quantum theory, and explain the relation to loop quantum gravity [3].

References

- [1] R. Penrose and W. Rindler, *Spinors and Space-Time, Two-Spinor Calculus and Relativistic Fields*, vol. 1. Cambridge University Press, Cambridge, 1984.
- [2] W. M. Wieland, *Hamiltonian spinfoam gravity*, *Class. Quantum Grav.* **31** (2014) 025002, arXiv:1301.5859.
- [3] C. Rovelli, *Quantum Gravity*, Cambridge University Press, Cambridge, November, 2008

The Matter Bounce Scenario in Loop Quantum Cosmology

The matter bounce scenario is an alternative to inflation where scale-invariant perturbations are generated in a dust-dominated contracting space-time. In the context of loop quantum cosmology, the cosmological singularity is generically resolved and replaced by a bounce; furthermore, it is possible to deterministically evolve the perturbations from the contracting branch through the quantum bounce and calculate the form of the perturbations at the onset of the expanding branch. For the matter bounce scenario in loop quantum cosmology, scale-invariant perturbations and a small tensor-to-scalar ratio are predicted [1], in agreement with the latest observations of Planck and BICEP2 [2].

References

- [1] E. Wilson-Ewing, *The Matter Bounce Scenario in Loop Quantum Cosmology*, JCAP 1303 (2013) 026, arXiv:1211.6269 [gr-qc].
- [2] Y.-F. Cai, J. Quintin, E. N. Saridakis and E. Wilson-Ewing, *Nonsingular bouncing cosmologies in light of BICEP2*, arXiv:1404.4364 [astro-ph.CO].

Plebanski sectors of the Lorentzian 4-simplex amplitude

The spin foam model is based on a BF-type action restricted by the simplicity constraint. However, the solutions to the simplicity constraint fall into five different sectors. In [1,2] it was argued that a certain mixing of these sectors and the freedom of choosing a tetrad orientation generates undesired terms in the asymptotic of the Euclidean EPRL-model and can be cured by an additional constraint. We here show that this is also the case for the Lorentzian model. This is joint work with J. Engle.

References

- [1] J. Engle, *A spin-foam vertex amplitude with the correct semiclassical limit*, Phys.Lett. B724 (2013) 333-337.
- [2] J. Engle, *A proposed proper EPRL vertex amplitude*, Phys.Rev D87 (2012) 084048.