Frontiers of Fundamental Physics 14

List of speakers in conference Frontiers of Fundamental Physics

Updated on January 8, 2015

Mikhail **Altaisky** (IKI) **July, 17, 15h00 – 15h30, Room 405, Frontiers of Fundamental Physics**

Continuous wavelet transform in quantum field theory

We describe the application of the continuous wavelet transform to calculation of the Green functions in quantum field theory: scalar ϕ^4 theory, quantum electrodynamics, quantum chromodynamics. The method of continuous wavelet transform in quantum field theory presented in [1,2] consists in substitution of the local fields $\phi(x)$ by those dependent on both the position x and the resolution a. The substitution of the action $S[\phi(x)]$ by the action $S[\phi_a(x)]$ makes the local theory into nonlocal one, and implies the causality conditions related to the scale a, the *region causality* [2]. These conditions make the Green functions $G(x_1, a_1, \dots, x_n, a_n) = \langle \phi_{a_1}(x_1) \dots \phi_{a_n}(x_n) \rangle$ finite for any given set of regions by means of an effective cutoff scale $A = \min(a_1, \dots, a_n)$.

References

[1] M.V.Altaisky, *Quantum field theory without divergences*, Phys. Rev. D 81(2010)125003

[2] M.V.Altaisky and N.E.Kaputkina, *Continuous wavelet transform in quantum field theory*, Phys. Rev. D 88(2013)025015

[3] J.D.Christensen and L. Crane, *Causal sites as quantum geometry*, J. Math. Phys. 46(2005)122502

Alessandro **Bisio** (UNIPV) **July, 17, 15h30 – 16h00, Room 405, Frontiers of Fundamental Physics**

From Quantum Cellular Automata to Quantum Field Theory

This talk is based on Refs. [1,2,3]. We explore the idea that Quantum Field Theory could be grounded on a Quantum Cellular Automaton (QCA) model (i.e. a discrete unitary evolution of a lattice of quantum systems). In the $1+1$-dimensional case, we show that the translation, parity and time-reversal symmetries lead to QCA model which recovers 1D Dirac field equation in the large scale limit. The discreteness of this model implies a breakdown of Lorentz covariance, which is replaced by a doubly special relativity model with an invariant lenght. Finally we will discuss the phenomenology of the QCA model and extensions to the three dimensional and interacting cases.

References

[1] A. Bisio, G. M. D'Ariano, A. Tosini *Quantum Field as a Quantum Cellular Automaton: the Dirac free evolution in one dimension* arXiv:1212.2839

[2] A. Bisio, G. M. D'Ariano, A. Tosini *Dirac quantum cellular automaton in one dimension: Zitterbewegung and scattering from potential* Phys. Rev. A **88**, 032301 (2013)

[3] A. Bibeau-Delisle, A. Bisio, G. M. D'Ariano, P. Perinotti, A. Tosini *Doubly-Special Relativity from Quantum Cellular Automata* arXiv:1310.6760

Alain **Blanchard** (IRAP) **July, 18, 14h30 – 15h00, Room 405, Frontiers of Fundamental Physics**

The cosmological constant and quantum vacuum

The acceleration of the expansion can now be regarded as established beyond reasonable doubt. However, the physical origin of this acceleration, the so-called dark energy, remains a puzzling problem in fundamental physics. The current explanations, quintessence and modified gravity, are based on a fundamental revision of known physics.

Here I will present a revival of an old proposition : that the origin of cosmic acceleration is due to the gravitational active vacuum.

Igor **Bulyzhenkov** (MIPT) **July, 17, 16h00 – 16h30, Room 405, Frontiers of Fundamental Physics**

Field and Matter or Pure Field Physics?

What did Einstein infer by stating [1] "Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. ... A thrown stone is, from this point of view, a changing field, where the states of greatest field intensity travel through space with the velocity of the stone. There would be no place, in our new physics, for both field and matter, field being the only reality."

One can indeed resolve the classical field equations in terms of infinite radial distributions for elementary masses and electrical charges [2]. Continuous field-type particles are counted by the left-hand side of the Einstein Equation where the Ricci scalar turns out to be geometrized mass density of such a non-empty, material space or inhomogeneous relativistic fluid [3]. Laboratory tests with precise clocks may distinguish in practice non-empty and empty space paradigms.

References

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[2] I. Bulyzhenkov, *Einstein's gravitation for Machian relativism of nonlocal energy-charges* , Int. J. Theor. Phys. 47 (2008) 1261-1269.
[3] I. Bulyzhenkov, *Densities of electron's continuum in gravitational and electromagnetic fields*, Bullet. Lebedev Phys. Inst. 41, N1 (2014) 1-5.

Alexander **Burinskii** (NSI RAS) **July, 15, 14h30 – 15h00, Room 405, Frontiers of Fundamental Physics**

From Kerr-Newman Black Hole to Spinning Particle: Where is There Hidden the Dirac Equation?

The Kerr-Newman black hole solution has many remarkable evidences of its relationships with the structure of the Dirac electron. Extra high spin/mass ratio of the electron corresponds to over-rotating Kerr geometry, singularity of which has to be regulated, forming a soliton-source [1]. There appears principal question: where and how the Dirac equation can be hidden inside the solitonic bag-like source of the Kerr-Newman spinning particle. In this talk we show that soliton represents a domain wall bubble confining the Higgs field in false vacuum state. The Kerr theorem creates two massless spinor fields (Weyl spinors) which interact via Yukawa coupling and turn into Dirac field, acquiring mass term from the Higgs field [3].

References

[1] A. Burinskii, *Regularized Kerr-Newman Solution as a Gravitating Soliton*,J. Phys. A: Math. Theor. **43**, 392001, (2010) [arXiv: 1003.2928].
[2] A. Burinskii, *What tells Gravity on the shape and size of an electron* Physics of Particles and Nuclei, **45** (1) (2014) 202. DOI:10.1134/S106377961401016X [arXiv:1212.2920].
[3] A. Burinskii, *Kerr-Newman electron as spinning soliton*, in: Proceedings of the "Advanced school on quantum foundations and open quantum systems" will be published in the Int. J. of Mod. Phys. A.

Valeriy **Dvoeglazov** (UAZ) **July, 16, 14h30 – 15h00, Room 405, Frontiers of Fundamental Physics**

Energy-Momentum Tensor in Electromagnetic Theory and Gravitation from Relativistic Quantum Equations

Recently, several discussions on the possible observability of 4-vector fields have been published in literature. Furthermore, several authors recently claimed existence of the helicity=0 fundamental field. We re-examine the theory of antisymmetric tensor fields and 4-vector potentials. We study the massless limits too. In fact, a theoretical motivation for this venture is the old papers of Ogievetskiĭ and Polubarinov, Hayashi, and Kalb and Ramond. Ogievetskii and Polubarinov proposed the concept of the *notoph*, whose helicity properties are complementary to those of the *photon*. We analyze the quantum field theory with taking into account mass dimensions of the notoph and the photon. It appears to be possible to describe both photon and notoph degrees of freedom on the basis of the modified Bargmann-Wigner formalism for the symmetric second-rank spinor. Next, we proceed to derive equations for the symmetric tensor of the second rank on the basis of the Bargmann-Wigner formalism in a straightforward way. The symmetric multispinor of the fourth rank is used. Due to serious problems with the interpretation of the results obtained on using the standard procedure we generalize it and obtain the spin-2 relativistic equations, which are consistent with the general relativity. Thus, in fact we deduced the gravitational field equations from relativistic quantum mechanics. The relations of this theory with the scalar-tensor theories of gravitation and $f(R)$ are discussed. Particular attention has been paid to the correct definitions of the energy-momentum tensor and other Nœther currents in the electromagnetic theory, the relativistic theory of gravitation, the general relativity, and their generalizations. We estimate possible interactions, fermion-notoph, graviton-notoph, photon-notoph, and conclude that they will be probably seen in experiments in the next few years.

Cohl **Furey** (PI) **July, 16, 15h00 – 15h30, Room 405, Frontiers of Fundamental Physics**

Charge Quantization from a Number Operator

In the early seventies, Günaydin and Gürsey discovered $SU_c(3)$ quark structure in the split octonions, [1]. Using their anti-commuting ladder operators, α_i, we show a direct route to a new $U(1)$ generator. This $U(1)$ generator behaves like electric charge, thereby allowing us to further identify states behaving like the electron and neutrino.

Our proposed electric charge turns out to be proportional to a number operator, consequently illuminating why it is quantized.

Using only this trio of ladder operators, and their conjugates, we construct a pair of *minimal left ideals*, which is shown to transform under $SU_c(3)$ and $U_{em}(1)$ as does a full generation of the standard model.

References
[1] M. Günaydin, F. Gürsey, *Quark Statistics and Octonions*, Phys. Rev. D, Vol. 9, No. 12 (1974)

Remo **Garattini** (UniBg) **July, 18, 15h00 – 15h30, Room 405, Frontiers of Fundamental Physics**

Distorting General Relativity: Gravity's Rainbow and f(R) theories at work

The talk is based on the results of Ref. [1], where we compute the Zero Point Energy in a spherically symmetric background combining the high energy distortion of Gravity's Rainbow [2] with the modification induced by a $f(R)$ theory. Here $f(R)$ is a generic analytic function of the Ricci curvature scalar R in 4D and in 3D. The explicit calculation is performed for a Schwarzschild metric [3]. Due to the spherically symmetric property of the Schwarzschild metric we can compare the effects of the modification induced by a $f(R)$ theory in 4D and in 3D. We find that the final effect of the combined theory is to have finite quantities that shift the Zero Point Energy. In this context we setup a Sturm-Liouville problem with the cosmological constant considered as the associated eigenvalue. The eigenvalue equation is a reformulation of the Wheeler-DeWitt equation which is analyzed by means of a variational approach based on gaussian trial functionals. With the help of a canonical decomposition, we find that the relevant contribution to one loop is given by the graviton quantum fluctuations around the given background. A final discussion on the connection of our result with the observed cosmological constant and the inflation problem is also reported.

References
[1] R. Garattini, *Distorting General Relativity: Gravity's Rainbow and $f(R)$ theories at work*, JCAP 1306 (2013) 017 arXiv:1210.7760 [gr-qc].
[2] J. Magueijo and L. Smolin, *Gravity's Rainbow,* Class. Quant. Grav. **21**, 1725 (2004) [arXiv:gr-qc/0305055].
[3] R. Garattini and G. Mandanici, *Modified Dispersion Relations lead to a finite Zero Point Gravitational Energy,* Phys. Rev. **D 83**, 084021 (2011); arXiv:1102.3803 [gr-qc].

Zbigniew **Haba** (ITFWroc) **July, 18, 15h30 – 16h00, Room 405, Frontiers of Fundamental Physics**

A fluid of diffusing particles and its cosmological behaviour

I. We discuss Einstein gravity for a fluid consisting of particles interacting with an environment of some other particles. The environment is described by a time-dependent cosmological term which is compensating the lack of the conservation law of the energy-momentum of the dissipative fluid. The dissipation is approximated by a relativistic diffusion in the phase space. We are interested in a homogeneous isotropic flat expanding Universe described by a scale factor a. At an early stage the particles are massless. We obtain explicit solutions of the diffusion equation for a fluid of massless particles at finite temperature. The solution is of the form of a modified Jüttner distribution with a time-dependent temperature. At later time Universe evolution is described as a diffusion at zero temperature with no equilibration. We find solutions of the diffusion equation at zero temperature which can be treated as a continuation to a later time of the finite temperature solutions describing an early stage of the Universe. The energy-momentum of the diffusing particles is defined by their phase space distribution. A conservation of the total energy-momentum determines the cosmological term up to a constant. The resulting energy-momentum inserted into Einstein equations gives a modified Friedmann equation. Solutions of the Friedmann equation depend on the initial value of the cosmological term. The large value of the cosmological constant implies an exponential expansion. If the initial conditions allow a power-like solution for a large time then it must be of the form $a \simeq \tau$ (no deceleration, τ is the cosmic time) . The final stage of the Universe evolution is described by a non-relativistic diffusion of a cold dust. The model is studied in [1]. The relativistic diffusion in [2-6].

II. As a second step we discuss extensions of the model I. We discuss interactions which lead to relativistic diffusion. In particular, we show that quantized electromagnetic fields and gravitons can lead to diffusion. Such a diffusion must be included on the rhs of Einstein equations if the quantized degrees of freedom are not treated explicitly as dynamical variables (some earlier work on the effect of gravitons is in [8,9]). Next, we consider the perturbation of the homogeneous metric of the model I in order to study the role of diffusion in the spreading of inhomogeneities at an early stage of the hot universe.

References

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Igor **Kanatchikov** (KCIK) **July, 15, 15h00 – 15h30, Room 405, Frontiers of Fundamental Physics**

Precanonical quantization from the first principles to quantization of gravity

I review mathematical structures of the De Donder-Weyl (DW) Hamiltonian theory for fields, which does not require space-time decompositions. Those structures: the polysymplectic form, Poisson-Gerstenhaber brackets of differential forms, which represent dynamical variables, the bracket form of the DW Hamiltonian field equations, and the fundamental brackets, allow us to formulate a generalization of canonical quantization to the DW framework. Because the structures of DW Hamiltonian theory are intermediate between the Lagrangian level and the canonical Hamiltonian level, and they don't require splitting into the space and time and don't consider the infinite dimensional spaces of field configurations, the corresponding approach is called precanonical quantization. I show how the quantization of fundamental brackets leads to a quantum theoretic formalism were the wave functions and operators are space-time Clifford algebra valued, and the role of the Schroedinger equation is played by a Dirac-like equation with the operator of the DW Hamiltonian function replacing the mass term. I briefly show how this approach allows to reproduce few simple results of quantum field theory and how the standard quantum field theory appears as a limiting case when the elementary volume parameter $1/\kappa$, which appears in precanonical quantization, tends to zero (more details will be given in the talk at the session on mathematical physics). After this review, I will show how the approach of precanonical quantization works for quantization of gravity in the first order vielbein - spin-connection formalism. It will include a generalization of DW hamiltonian theory to the singular case, its quantization, formulation of the generalized Schroedinger equation for the wave function on the bundle of spin-connection coefficients over space-time, and the definition of the appropriate Hilbert space. In this framework the quantum space-time is discussed by the transition amplitudes between the values of spin-connection at different points of space-time, which can be determined from the equations of the theory. I will also discuss simple applications of the results in the context of quantum cosmology.

References

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[3] I.V. Kanatchikov, *On the precanonical structure of the Schrödinger wave functional,* arXiv:1312.4518; *Precanonical Quantization and the Schrödinger wave functional revisited,* arXiv:1112.5801, to appear in Adv. Theor. Math. Phys.
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Joseph **Kouneiher** (UNS) **July, 16, 16h00 – 16h30, Room 405, Frontiers of Fundamental Physics**

Higgs fields, Yang-Mills Quantized theories and the Cohomological origine of the mass

In this talk we want to show how the mass can enter Yang-Mills theories through central (pseudo) extensions of the corresponding gauge group. This mechanism does not involve extra (Higgs) scalar particles and could provide new clues for the better understanding of the nature of the Symmetry Breaking Mechanism. In this talk we are going to outline the essential points of this approach.

Marc **Lachièze-Rey** (APC) **July, 17, 14h30 – 15h00, Room 405, Frontiers of Fundamental Physics**

Dynamics of histories

We propose an unified framework, based on the notion of histories, which applies to time dynamics and to field theories. It accepts a Lagrangian and a Hamiltonian formulations, both entirely covariant in the case of field theories. Dynamical equations and conservation laws take a very simple universal expression. Developing differential calculus in the (infinite dimensional) space of histories, we define a generalized symplectic form. It leads very naturally to the multisymplectic formalism in field theories (the usual symplectic form for time dynamics appearing as a particular case) and it provides the usual on-shell symplectic form (in the space of solutions). We apply to first order canonical general relativity.

Sergey **Mayburov** (FIAN) **July, 15, 16h00 – 16h30, Room 405, Frontiers of Fundamental Physics**

Fuzzy Topology, Quantization and Gauge Fields

It was argued earlier that Dodson-Zeeman fuzzy topology (FT)[1] represents the possible mathematical basis for quantum space-time structure[2]. Here the quantization formalism related to it will be described[3,4].

As the example, the quantization of massive particles is considered, it's shown that the coordinate uncertainty is generic in FT. FT fundamental set D is Poset[3,4], so that some its element pairs in place of standard ordering relation $d_j \leq d_k$, can obey to incomparability relation: $d_l \sim d_m$. For illustration, consider discrete Poset $D = A^p \cup B$, which includes the subset of incomparable elements $A^p = \{a_j\}$, and the subset $B = \{b_i\}$ which is maximal totally ordered D subset. B indexes grow correspondingly to their ordering, *i.e.* $\forall\, i,\ b_i \leq b_{i+1}$. Suppose that for some a_j and B interval $\{b_l, b_n\}$, $a_j \sim b_i\, ; \forall i:\ l \leq i \leq n$. In this case a_j is "smeared" over $\{b_l, b_n\}$ interval, which is analogue of a_j coordinate uncertainty, if to regard B as D "coordinate axe". Analogously to it, 1-dimensional model Universe corresponds to Poset $U = A^p \cup X$ where A^p is the massive particle subset, X-continuous ordered subset R^1, which describes 1-dimensional euclidian geometry. If for some a_j and X interval $\{x_c, x_d\}$ the relation $a_j \sim x_b$ holds for all $x_b \in \{x_c, x_d\}$, then a_j possess x-coordinate uncertainty of the order $|x_d - x_c|$. To detalize a_j characteristics, the corresponding fuzzy weight $w_j(x) \geq 0$ introduced with the norm $\|w_j\| = 1$, so that $w_j(x)$ value indicates where on X axe a_j is mainly concentrated[3]. In this framework a_j corresponds to the formal definition of fuzzy point and U of fuzzy set[3].

In such approach massive particle m can be described as the evolving fuzzy point $a_i(t)$ of U. It's shown then that the corresponding normalized m density $w(x,t)$ evolves according to the flow continuity equation: $\frac{\partial w}{\partial t} = -\frac{\partial (wv)}{\partial x}$ where $v(x,t)$ is w flow local velocity. The independent m parameters $w(x), v(x)$, which characterize m state, can be unambiguously mapped to normalized complex function $\varphi(x)$. Assuming space-time shift invariance, it's proved that $\varphi(x,t)$ evolution obeys to Schrödinger equation for arbitrary m mass μ, such theory can be also extended for 3-dimensional case[4]. It's proved also that in relativistic case m evolution described by Dirac equation for spin $\frac{1}{2}$. Particle's interactions on such fuzzy manifold are shown to be gauge invariant, the interactions of fermion muliplets are performed by Yang-Mills fields[5].

References
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Vishagan **Sivanesan** (Iap) **July, 18, 14h30 – 15h00, Amphi "Sciences Naturelles", Cosmology**

A No Go Theorem for Gallileon like "Odd P-Forms"

We explore the possibility for generalized gauge invariant p-form theories on flat space-time. 'Galileons' are a well known example of scalar fields (0-form) that has non-linear extension of the second derivative terms in field equations. We prove that there is an obstruction to extending to non-linear order in the second derivatives for gauge invariant p-forms when p is odd. That is the equations of motion for the p-form field are at most linear in the second derivative of the field.

Alessandro **Spallicci** (UO, OSUC) **July, 15, 15h30 – 16h00, Room 405, Frontiers of Fundamental Physics**

Solar wind test of the de Broglie-Proca's massive photon with Cluster multi-spacecraft data

The exam of the literature on the large-scale astrophysical limits for the photon mass inspires a critical attitude and prompts to question whether these limits are nothing more than the outcome of idealized models. This view is confirmed by Goldhaber and Nieto [1] who state "Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterizations. This is perhaps due to the temptation to assert too strongly something one 'knows' to be true. A look at the summary of the Particle Data Group [...] hints at this."

We use spacecraft data in the solar wind at 1 AU to estimate the mass upper limit of the de Broglie-Proca's photon, by looking for deviations from the Ampère's law. We take advantage of the Cluster spacecraft which both allow the direct computation of $\nabla \times \vec{B}$ from simultaneous four-point measurements of the magnetic field and provide measurements of particle currents. We estimate the upper bound for the mass m_γ to be 1.4×10^{-49} kg without using any *ad hoc* model. Finally, we discuss how this limit can be lowered and compare with currently accepted values in the solar wind.

References
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Alessandro **Tosini** (UNIPV) **July, 15, 17h00 – 17h30, Room 405, Frontiers of Fundamental Physics**

Informational features of Fermionic systems

The anticommutation of Fermionic fields raises the problem of simulating the evolution of Fermionic systems by means of commuting quantum systems, say qubits. We tackle [1,2] the issue of retaining locality of Fermionic operations considering local Fermionic modes as the elementary systems of an operational probabilistic theory. The locality of Fermionic operations, namely operations on systems that are not causally connected must commute, implies the parity superselection rule that inhibits the superposition of states with an even and an odd number of excitations. As a result the Fermionic theory lacks two distinctive traits of quantum theory, the local tomography and the monogamy of the entanglement.

References
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John **Williamson** (UoG) **July, 16, 15h30 – 16h00, Room 405, Frontiers of Fundamental Physics**

A new theory of light and matter

A new, linear field theory of light and matter is proposed, sharpening the principle of relativity and adding four new coupled differential equations to the original four of Maxwell. The new theory, though continuous, allows only propagating pure field solutions with an integral angular momentum. These are identified with the photon. Solutions with rest-mass are necessarily charged, with a value close to the elementary charge [1]. They have a double-covering rotation in momentum space with a resulting half-integral spin and statistics, as was discussed on the basis of an earlier non-linear theory [2]. The result is that, just as was the case for de Broglie's concept of the harmony of phases [3] which led to his famous relation and the subsequent development of quantum mechanics, it is a rigorous consideration of the principle of relativity that leads to the quantisation of light and matter.

References
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