Frontiers of Fundamental Physics 14
List of speakers in conference
Mathematical Physics

Updated on January 8, 2015

Paolo **Aschieri** (UPO) **July, 17, 17h00 – 17h45, Room 406, Mathematical Physics**

Deformation quantization of Noncommutative Principal Bundles

Drinfeld twist deformation theory of modules and algebras that carry a representation of a Hopf Algebra H can be extended to deform also morphisms and connections that are not H-equivariant. In this talk I present how similar techniques allow to canonically deform principal G-bundles, and in general how Hopf-Galois extensions are canonically deformed to new Hopf-Galois extensions.

Twisting the structure group we obtain principal bundles with noncommutative fiber and where the structure group is a quantum group. Twisting the automorphism group of the principal bundle we further obtain a noncommutative base space.

Fabien **Besnard** (EPF) **July, 15, 16h00 – 16h30, Room 406, Mathematical Physics**

Causality and Noncommutative Geometry

Motivated by the introduction of causality in noncommutative geometry, we define the notion of *isocone*. An isocone is a closed convex cone in a C^*-algebra, containing the unit, which separates the states and is stable by non-decreasing continuous functional calculus.

We show that our definition is physically well-motivated, and corresponds exactly to the structure of non-decreasing real functions on a (compact) topological ordered set satisfying a natural compatibility condition between the topology and the partial order, when the C^*-algebra is commutative [1].

We also give the complete classification of isocones in finite dimensional algebras, corresponding to finite noncommutative ordered spaces, and give some examples in infinite dimension [2].

Finally we show that the existence of an isocone on an almost commutative algebra $\mathcal{C}(M) \otimes A_F$ of the kind which appears in the NCG formulation of the Standard Model forces the causal order relation on M to disappear in the neighbourhood of every point [3]. Although the scale at which causality disappears is left unspecified by this very general mathematical result, it only depends on the noncommutativity of the algebra A_F, hence on particle physics, and might thus be expected to be much larger than the Planck scale and to possibly leave an observable imprint on the cosmic microwave background.

References
[1] F. Besnard, *A noncommutative view on geometry and order*, J. Geom. Phys. **59**, pp. 861–875 (2009) abs/0804.3551
[2] F. Besnard, *Noncommutative ordered spaces : examples and counterexamples*, submitted to J. Geom. Phys., abs/1312.2442
[3] N. Bizi, F. Besnard, work in progress

Pierre **Bieliavsky** (UCLouvain) **July, 17, 14h30 – 15h15, Room 406, Mathematical Physics**

On Drinfel'd twists and their use in non-commutative geometry

I will present some recent developments in the theory of non-formal Drinfel's twists. A Drinfel'd twist consists in a tool that allows to deform in an associative way any associative algebra that possesses a given symmetry. For instance the Moyal twist is a Drinfel'd twist for the abelian symmetry $\mathbb{R}^d$. I will report a general method for constructing Drinfel'd twists based on generally non-abelian symmetries. I will conclude by mentioning applications in various domains.

Christian **Brouder** (IMPMC) **July, 15, 15h15 – 16h00, Room 406, Mathematical Physics**

Noncommutative version of Borcherds' approach to quantum field theory

Richard Borcherds recently proposed [1] an elegant geometric version of renormalized perturbative quantum field theory in curved spacetimes, where Lagrangians are sections of a Hopf algebra bundle over a smooth manifold. Borcherds' framework provides an algebraic interpretation of causal perturbation theory (Stueckelberg, Bogoliubov, Epstein, Glaser...). We present a noncommutative version of Borcherds approach. Although the normal product is now the tensor product, this version can still be equivalent to standard quantum field theory if the Hopf algebra fiber is graded cocommutative [2]. The relation with almost-commutative geometry [3] is discussed.

References
[1] R. E. Borcherds, *Renormalization and quantum field theory*, Algebra & Number Theory 5 (2011) 627.
[2] Ch. Brouder and F. Patras, *Nonlocal, noncommutatie diagrammatics and the linked cluster theorems*, J. Math. Chem. 50 (2012) 552.
[3] A. Connes and M. Marcolli, *Noncommutative Geometry, Quantum Fields and Motives*, American Mathematical Society, Providence, USA. (2008).

Sylvain **Carrozza** (CPT) **July, 18, 16h00 – 16h30, Room 406, Mathematical Physics**

Renormalization in Tensorial Group Field Theories

In this talk, I will review some recent results about the renormalization of Tensorial Group Field Theories. These theories are motivated by an approach to quantum gravity which lies at the crossroad of tensor models and loop quantum gravity. From the mathematical point of view, they are quantum field theories defined on compact Lie groups, with specific non-local interactions. Interestingly, these non-localities can be controlled and several models have now been proven perturbatively well-defined [1, 2, 3, 4, 5, 6]. I will focus on a $SU(2)$ model inspired by Eucliden $3d$ quantum gravity, which has been proven renormalizable at all orders with up to φ^6 interactions [6]. Time allowing, I will also present new results about the renormalization group flow of this model.

References

[1] J. Ben Geloun and V. Rivasseau, "A Renormalizable 4-Dimensional Tensor Field Theory," Commun. Math. Phys. **318**, 69 (2013) [arXiv:1111.4997 [hep-th]].
[2] J. Ben Geloun and D. O. Samary, "3D Tensor Field Theory: Renormalization and One-loop β-functions," Annales Henri Poincare **14**, 1599 (2013) [arXiv:1201.0176 [hep-th]].
[3] J. Ben Geloun and E. R. Livine, "Some classes of renormalizable tensor models," J. Math. Phys. **54**, 082303 (2013) [arXiv:1207.0416 [hep-th]].
[4] S. Carrozza, D. Oriti and V. Rivasseau, "Renormalization of Tensorial Group Field Theories: Abelian $U(1)$ Models in Four Dimensions," Commun. Math. Phys. **327**, 603 (2014) [arXiv:1207.6734 [hep-th]].
[5] D. O. Samary and F. Vignes-Tourneret, "Just Renormalizable TGFT's on $U(1)^d$ with Gauge Invariance," Commun. Math. Phys. (2014) [arXiv:1211.2618 [hep-th]].
[6] S. Carrozza, D. Oriti and V. Rivasseau, "Renormalization of an $SU(2)$ Tensorial Group Field Theory in Three Dimensions," Commun. Math. Phys. (2014) [arXiv:1303.6772 [hep-th]].

Ali **Chamseddine** (AUB and IHES) **July, 16, 14h30 – 15h15, Room 406, Mathematical Physics**

Noncommutative Geometry and Physics

Classification of finite noncommutative spaces points uniquely to the standard model provided the connection is linear. Relaxing the constraint of linearity allows for the Pati-Salam model of unification of quarks and leptons.

Francesco **D'Andrea** (UNINA) **July, 15, 17h00 – 17h45, Room 406, Mathematical Physics**

Vector bundles on the noncommutative torus from cochain quantization

The non-commutative torus is commonly described as a cocycle quantization of the group (C^*-)algebra of the abelian group $\mathbb{Z}^2$. In the first part of the talk I will explain how, using the WBZ transform of solid state physics, finitely generated projective modules over the noncommutative torus can be interpreted as deformations of vector bundles on elliptic curves by the action of a 2-cocycle, provided that the deformation parameter of the NC-torus and the modular parameter of the elliptic curve satisfy a non-trivial relation.

I will then discuss the relation between (formal) deformations of vector bundles on the torus and cochain twists based on the Lie algebra of the 3-dimensional Heisenberg group.

Based on a joint work with G. Fiore and D. Franco.

Michal **Eckstein** (UJ) **July, 16, 16h00 – 16h30, Room 406, Mathematical Physics**

Causal structure for noncommutative geometry

Noncommutative Geometry à la Connes offers a promising framework for models of fundamental interactions. To guarantee the correct signature, the theory of Lorentzian spectral triples has been developed. I will briefly summarise its main elements and show that it can accommodate a sensible notion of causality understood as a partial order relation on the space of states on an algebra [1]. For almost-commutative algebras of the form $\mathcal{C}^\infty(M)\otimes\mathcal{A}_F$, with $\mathcal{A}_F$ being finite-dimensional, the space of (pure) states is a simple product of space-time M and an internal space. The exploration of causal structures in this context leads to a surprising conclusion [2]: The motion in both space-time and internal space is restricted by a "finite speed of light" constraint. I will explain the latter on 2 simple toy-models.

References:

[1] N. Franco, M. Eckstein: *An algebraic formulation of causality for noncommutative geometry*, Classical and Quantum Gravity **30** (2013) 135007.
[2] N. Franco, M. Eckstein: *Exploring the Causal Structures of Almost Commutative Geometries*, SIGMA **10** (2014) 010.

Patrick **Iglesias-Zemmour** (I2M, CNRS) **July, 17, 15h45 – 16h30, Room 406, Mathematical Physics**

Symplectic Diffeology and Moment Maps

I shall discuss the general framework of symplectic diffeology through examples, in infinite dimension and in singular context: infinite projective space, orbifolds, symplectic irrational tori etc. I will discuss how the condition to be symplectic in diffeology involves at the same time the universal moment map, relative to the automorphisms of the structure, and the decomposition of the space into orbits under the automorphisms. I will also discuss a family of examples that mixes infinite dimension and singular quotients: the infinite quasi-projective spaces.

That will show how diffeology is an operable theory and handles the objects of mathematical physics in a simple but rigorous way.

Gandalf **Lechner** (ITP Leipzig) **July, 18, 17h00 – 17h45, Room 406, Mathematical Physics**

Hyperbolic PDEs with non-commutative time

In this talk, I will report on joint work with Rainer Verch [1] on hyperbolic PDEs with non-commutative time, i.e. linear integro-differential equations of the form $(D + \lambda W)f = 0$, where D is a (pre-)normal hyperbolic differential operator on $\mathbb{R}^n$, $\lambda \in \mathbb{C}$ is a coupling constant, and W a regular integral operator which is non-local in time, so that a Hamiltonian formulation is not possible. Such equations appear in the context of wave or Dirac equations on non-commutative deformations of Minkowski space. It will be discussed that at small coupling, the hyperbolic character of D is essentially preserved, unique advanced/retarded fundamental solutions can be constructed, and the acausal behavior of the solutions is well-controlled. Although the Cauchy problem is ill-posed in general, a scattering operator can be calculated which describes the effect of W on the space of solutions of D.

It is also described how these results can be used for the analysis of classical and quantum field theories on non-commutative spaces.

References
[1] G. Lechner and R. Verch, *Linear hyperbolic PDEs with non-commutative time*, Preprint, arXiv:1307.1780

Fedele **Lizzi** (UNINA) **July, 15, 14h30 – 15h15, Room 406, Mathematical Physics**

Noncommutative Geometry, the Spectral Action and Fundamental Symmetries

Noncommutative Geometry, i.e., the spectral data of generalized spaces, provides a fruitful approach to the standard model of fundamental interactions. This is done via the spectral action, which is a function of the Dirac operator. It is a regularized trace, cutoff at a scale. This cutoff is stye point in which all gauge interactions are equally strong, and it may represent a phase transition of the theory to a pre geometric phase. I will discuss the role of this field theory cutoff, and the symmetries and structure of space-time that one can infer from the spectral action.

References
[1] M. Anselmino, A. Efremov and E. Leader, *The theory and phenomenology of polarized deep inelastic scattering*, ys. Rept. 261 (1995) 1 [Erratum ibid 281 (1997) 399] [hep-ph/9501369].
[2] R. Penrose and W. Rindler, Spinors and Space-time, Vol. 2: Spinor and twistormethods in space-time geometry, Cambridge University Press, Cambridge U.K. (1986), pg. 501.
[3] CMS collaboration, Technical Design Report Vol. 1, CERN-LHCC-2006-001.

Pierre **Martinetti** (UNINA) **July, 15, 17h45 – 18h30, Room 406, Mathematical Physics**

Geometry with a cut-off

Various aspects of fundamental physics – like renormalization in quantum field theory or some considerations about quantum gravity – point towards the existence of a cut-off in either the momentum or the position space. As a consequence, usual geometrical notions such as points and geodesic distance loose their meaning. We will show how non-commutative geometry provide interesting tools to study the geometry of such “cut-off spaces”, including topological and metric aspects.

Jean-Philippe **Michel** (ULg) **July, 17, 15h15 – 15h45, Room 406, Mathematical Physics**

Higher Symmetries of Laplace and Dirac operators - towards supersymmetries

In this talk, I will survey some of the results obtained in [3, 4, 5]. On a flat (pseudo-)Riemannian manifold, the higher symmetries of the Laplacian form an associative algebra of differential operators, determined by Eastwood in [1]. It plays a central role in higher spin field theory and give a geometric realization of a highly non-trivial object from Lie theory: the Joseph ideal [2]. Using quantization methods, I propose a simple proof of Eastwood’s result and extend it to the system Laplace $\oplus$ Dirac operators. In dimension 4, its higher symmetries are generated by the conformal supersymmetries discovered by Wess and Zumino [6].

References
[1] M. G. Eastwood. *Higher symmetries of the Laplacian.* Ann. of Math. (2), 161(3):1645–1665, 2005.
[2] A. Joseph. *The minimal orbit in a simple Lie algebra and its associated maximal ideal.* Ann. Sci. École Norm. Sup. (4), 9(1):1–29, 1976.
[3] J.-Ph. Michel. *Higher symmetries of Laplacian via quantization.* Ann. Inst. Fourier (to appear).
[4] J.-Ph. Michel, F. Radoux, and J. Šilhan. *Second order symmetries of the conformal Laplacian.* SIGMA, 10:Paper 016, 2014.
[5] J.-Ph. Michel and J. Šilhan. *Higher symmetries of Laplace and Dirac operators.* In preparation.
[6] J. Wess and B. Zumino. *Supergauge transformations in four dimensions.* Nucl. Phys. B, 70(1):39 – 50, 1974.

Nicola **Pinamonti** (Univ. of Genova) **July, 16, 17h00 – 17h45, Room 406, Mathematical Physics**

Influence of quantum matter fluctuations on the expansion parameter of timelike geodesics

During this talk, we shall discuss the passive influence of quantum matter fluctuations on the expansion parameter of a congruence of timelike geodesics in a semiclassical regime. In particular, we shall see that the perturbations of this parameter can be considered to be elements of the algebra of matter fields at all perturbative order. Hence, once a quantum state for matter is chosen, it is possible to explicitly evaluate the amplitude of the geometric fluctuations. After introducing the formalism necessary to treat similar problems, we estimate the approximated probability of having a geodesic collapse in a flat spacetime due to those fluctuations. Starting from this, some estimate of the spacetime uncertainty relations will be given.

References

[1] N. Drago, N. Pinamonti *"Influence of quantum matter fluctuations on geodesic deviation"*, Preprint February (2014) [1402.4265 [math-ph].

[2] N. Pinamonti, D. Siemssen *"Global Existence of Solutions of the Semiclassical Einstein Equation in Cosmological Spacetime"*, Commun. Math. Phys. Accepted for publication. (2014) [1309.6303 [math-ph].

[3] N. Pinamonti, *"On the initial conditions and solutions of the semiclassical Einstein equations in a cosmological scenario"*, Commun. Math. Phys. **305** (2011) 563-604.

Kasia **Rejzner** (York) **July, 18, 15h15 – 16h00, Room 406, Mathematical Physics**

Perturbative algebraic QFT as a universal framework for constructing physically motivated models in quantum field theory

Perturbative algebraic quantum field theory is a formalism which allows to put perturbative QFT on a solid mathematical basis and solves many conceptual problems. It has proven to be a very successful framework for QFT on curved spacetimes, since it allows to separate the algebraic structure of the theory from the construction of a state. The main idea, inspired by the Haag-Kastler axiomatic framework, is to define a model of a QFT by giving a net of unital $*$-algebras, assigned to regions of spacetime. To construct such a model, one starts with a free classical theory, then obtains the free quantum theory via deformation quantization and finally introduces the interaction by means of Epstein-Glaser renormalization. In this overview talk I will show how this method works in particular examples and I will report on recent results.

Christoph **Stephan** (UP) **July, 16, 17h45 – 18h30, Room 406, Mathematical Physics**

Model building in almost-commutative geometry

Alain Connes' noncommutative geometry allows to unify the classical Yang-Mills-Higgs theory and General relativity in a single geometrical framework, so called almost-commutative geometries. This unification implies restrictions for the couplings of the Standard Model at a given cut-off energy which reduce the degrees of freedom compared to the classical Standard Model.

I will give an introduction to the basic ideas of almost-commutative model building and present models beyond the Standard Model that may be phenomenologically interesting. These models include extensions of the fermionic and the gauge sector as well as extensions of the scalar sector.

Guo Chuan **Thiang** (Univ. of Oxford, Math. Institute) **July, 17, 17h45 – 18h15, Room 406, Mathematical Physics**

On the K-theoretic classification of topological phases of matter

In recent years, there has been a lot of interest in studying the topological phases of quantum matter. A K-theoretic approach was suggested by Kitaev, who produced a Periodic Table of topological insulators and superconductors [2]. We take the algebraic viewpoint, and study the gapped topological phases of free fermions through a twisted crossed product C^*-algebra associated to the symmetry data of the system. Allowing for projective unitary-antiunitary representations, in the sense of Wigner, as well as charge-conjugation symmetries, leads to a $\mathbb{Z}_2$-graded real twisted group C^*-algebra, which completely encodes all the symmetry data of a quantum system. We define two K-theory-type invariants of this algebra: the super-representation group classifies symmetry-compatible gapped phases, while the K-theoretic difference-group classifies differences between stable homotopy classes of such phases. We also provide a consistent physical interpretation of these classification groups, which appears to vary between existing treatments in the literature. Our approach generalises, to the non-commutative setting, the twisted K-theory approach of Freed and Moore [1]. It has the advantage of treating all symmetries on an equal footing, and powerful results from the K-theory of crossed products are available. We recover Kitaev's Periodic Table as a special case, and clarify the origin of the periodicities and "dimension-shifts" in his table.

References

[1] Freed, D.S. and Moore, G.W., *Twisted Equivariant Matter*, Ann. Henri Poincaré, 14(8), pp. 1927–2023 (2013)

[2] Kitaev, A., *Periodic table for topological insulators and superconductors*, arXiv preprint arXiv:0901.2686 (2009)

Walter **van Suijlekom** (RU) **July, 16, 15h15 – 16h00, Room 406, Mathematical Physics**

Inner perturbations in noncommutative geometry

Starting with an algebra, we define a semigroup which extends the group of invertible elements in that algebra. As we will explain, this semigroup describes inner perturbations of noncommutative manifolds, and has applications to gauge theories in physics. We will present some elementary examples of the semigroup associated to matrix algebras, and to (smooth) functions on a manifold. Joint work with Ali Chamseddine and Alain Connes.

References

[1] A.H. Chamseddine, A. Connes, and W. D. Van Suijlekom. "Inner fluctuations in noncommutative geometry without the first order condition", *J. Geom. Phys.* 73 (2013) 222–234.
[2] A.H. Chamseddine, A. Connes, and W. D. van Suijlekom. "Beyond the spectral Standard Model: Emergence of Pati-Salam unification", *JHEP* 1311 (2013) 132.

Raimar **Wulkenhaar** (WWU) **July, 18, 14h30 – 15h15, Room 406, Mathematical Physics**

Construction of a quantum field theory in four dimensions

We prove that the $\lambda\phi^4_4$ quantum field theory on noncommutative Moyal space is, in the limit of infinite noncommutativity, exactly solvable in terms of the solution of a non-linear integral equation. Surprisingly, this limit describes Schwinger functions of a Euclidean quantum field theory on standard $\mathbb{R}^4$ which satisfy the easy Osterwalder-Schrader axioms boundedness, covariance and symmetry. We prove that the decisive reflection positivity axiom is, for the 2-point function, equivalent to the question whether or not the solution of the integral equation is a Stieltjes function. The numerical solution of the integral equation leaves no doubt that this is true for coupling constants $\lambda \in [-0.39, 0]$.